Rigidity, Boundary Interpolation and Reproducing Kernels

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Recall that a Schur function is a function analytic in the open unit disk and bounded by one in modulus there. When the angular convergence is replaced by the unrestricted one, the following rigidity result is due to D. M. Burns and S. G. Krantz [2].

Theorem 1 *Assume that a Schur function s satisfies*

$$s(z) = z + O((1-z)^4), \quad z \hat{\to} 1, \tag{1}$$

where $\hat{\to}$ denotes angular convergence. Then $s(z) \equiv z$.

We use reproducing kernel methods, and in particular the results on boundary interpolation for generalized Schur functions proved in [1] to prove a general rigidity theorem which extend this result. The methods and setting allow us to consider the non-positive case. For instance we have the following result, which seems to be the first rigidity result proved for functions with poles.

Theorem 2 *Let s be a generalized Schur function with one negative square and assume that*

$$s(z) - \frac{1}{z} = O((1-z)^4), \quad z \hat{\to} 1.$$

Then

$$s(z) \equiv \frac{1}{z}.$$

Details can be found in a manuscript on the arxiv site.

References

[1] D. Alpay, A. Dijksma, H. Langer, and G. Wanjala, Basic boundary interpolation for generalized Schur functions and factorization of rational J–unitary matrix functions, Operator Theory: Advances and Applications, volume 165, pages 1–29, Birkhäuser Verlag, Basel, 2006.

[2] D. M. Burns and S. G. Krantz, Rigidity of holomorphic mappings and a new Schwarz lemma at the boundary, *J. Amer. Math. Soc.* 7 (1994), 661–676.