Iterates of the Schur Class Operator-Valued Function and Their Conservative Realizations

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Let $\mathfrak{M}$ and $\mathfrak{N}$ be separable Hilbert spaces and let $\Theta(z)$ be the function from the Schur class $\mathbf{S}(\mathfrak{M},\mathfrak{N})$ of contractive functions holomorphic on the unit disk. The operator generalization of the Schur algorithm associates with Θ the sequence of contractions (the Schur parameters of Θ)

$$\Gamma_0 = \Theta(0) \in L(\mathfrak{M},\mathfrak{N}),\ \Gamma_n \in L(\mathfrak{D}_{\Gamma_{n-1}}, \mathfrak{D}_{\Gamma^*_{n-1}})$$

and the sequence of functions $\Theta_0 = \Theta$, $\Theta_n \in \mathbf{S}(\mathfrak{D}_{\Gamma_n}, \mathfrak{D}_{\Gamma^*_n})$ $n = 1, \ldots$ connected by the relations

$$\Gamma_n = \Theta_n(0),\ \Theta_n(z) = \Gamma_n + zD_{\Gamma^*_n}\Theta_{n+1}(z)(I + z\Gamma^*_n\Theta_{n+1}(z))^{-1}D_{\Gamma_n},\ |z| < 1.$$

The function $\Theta_n(z)$ is called the n-th Schur iterate of Θ.

The function $\Theta(z) \in \mathbf{S}(\mathfrak{M},\mathfrak{N})$ can be realized as the transfer function $\Theta(z) = D + zC(I - zA)^{-1}B$ of a linear conservative and simple discrete-time system

$$\tau = \left\{ \begin{bmatrix} D & C \\ B & A \end{bmatrix}; \mathfrak{M}, \mathfrak{N}, \mathfrak{H} \right\}$$

with the state space $\mathfrak{H}$ and the input and output spaces $\mathfrak{M}$ and $\mathfrak{N}$, respectively.

In this talk we give a construction of conservative and simple realizations of the Schur iterates Θ_n by means of the conservative and simple realization of Θ.