On the Uniform Convergence of Pade Approximants for a Class of Definitizable Functions

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joint work with M. Derevyagin and P. Jonas

Let us say that a function ψ meromorphic in $\mathbb{C}_+$ belongs to the class $\mathbf{D}_{\kappa,-\infty}$ ($\kappa \in \mathbb{Z}_+$) if $\psi(\lambda)/\lambda$ belongs to the generalized Nevanlinna class $\mathbf{N}_\kappa$ and for some $s_j \in \mathbb{R}$ ($j \in \mathbb{Z}_+$) the following asymptotic expansion holds:

$$\psi(\lambda) = -\frac{s_0}{\lambda} - \frac{s_1}{\lambda^2} - \cdots - \frac{s_{2n}}{\lambda^{2n+1}} - \cdots \quad (\lambda \widehat{\to} \infty).$$

It is shown that for every $\psi \in \mathbf{D}_{\kappa,-\infty}$ there is a subsequence of diagonal Pade approximants, which converges to ψ locally uniformly on $\mathbb{C} \setminus \mathbb{R}$ in spherical metric. Conditions for the convergence of this subsequence on the real line are also found.