When are the Products of two Normal Operators Normal?

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Given two normal operators A and B on a Hilbert space it is known that, in general, AB is not normal. Even more, I. Kaplansky had shown that it may be possible that AB is normal while BA is not. In this paper we address the question on (spectral) characterizations of those pairs of normal operators A and B for which both the products AB and BA are normal. This question has been solved for finite dimensional spaces by F.R. Gantmaher and M.G. Krein in 1930, and for compact normal operators A and B by N.A. Wiegmann in 1949. Actually, in these cases, the normality of AB is equivalent with that of BA. We consider the general case (no compactness assumption) by means of the Spectral Multiplicity Theorem for normal operators in the von Neumann's direct integral representation and the technique of integration/disintegration of Borel measures on metric spaces.