Projective Hilbert Space Structures at Exceptional Points and Krein Space Related Boost Deformations of Bloch Spheres

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Simple non-Hermitian quantum mechanical matrix toy models are considered in the parameter space vicinity of Jordan-block structures of their Hamiltonians and corresponding exceptional points of their spectra. In the first part of the talk, the operator (matrix) perturbation schemes related to root-vector expansions and expansions in terms of eigenvectors for diagonal spectral decompositions are projectively unified and shown to live on different affine charts of a dimensionally extended projective Hilbert space. The monodromy properties (geometric or Berry phases) of the eigenvectors in the parameter space vicinities of spectral branch points (exceptional points) are briefly discussed.

In the second part of the talk, it is demonstrated that the recently proposed $\mathcal{PT}-$symmetric quantum brachistochrone solution [C. Bender et al, Phys. Rev. Lett. **98**, (2007), 040403, quant-ph/0609032] has its origin in a mapping artifact of the $\mathcal{PT}-$symmetric 2×2 matrix Hamiltonian in the vicinity of an exceptional point. Over the brachistochrone solution the mapping between the $\mathcal{PT}-$symmetric Hamiltonian as self-adjoint operator in a Krein space and its associated Hermitian Hamiltonian as self-adjoint operator in a Hilbert space becomes singular and yields the physical artifact of a vanishing passage time between orthogonal states. The geometrical aspects of this mapping are clarified with the help of a related hyperbolic Möbius transformation (contraction/dilation boost) of the Bloch (Riemann) sphere of the qubit eigenstates of the 2×2 matrix model.

The controversial discussion on the physics of the brachistochrone solution is briefly commented and a possible resolution of the apparent inconsistencies

is sketched.

partially based on:
J. Phys. A 40 (2007) 8815-8833; arXiv:0704.1291 [math-ph].
arXiv:0709.0483 [quant-ph].