The Similarity Problem for J-Nonnegative Sturm-Liouville Operators

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We present new sufficient conditions for the similarity of J-self-adjoint Sturm-Liouville operators to self-adjoint ones. These conditions are formulated in terms of Weyl-Titchmarsh m-coefficients. This result is exploit to prove the regularity of the critical point zero for various classes of J-nonnegative Sturm-Liouville operators. In particular, we prove that 0 is a regular critical point of

$$A = (\operatorname{sgn} x)(-d^2/dx^2 + q(x))$$

if $q \in L^1(\mathbb{R}, (1 + |x|)dx)$. Moreover, in this case A is similar to a self-adjoint operator if and only if it is J-nonnegative. We also show that the latter condition on q is sharp, that is we construct a potential $q_0 \in \cap_{\gamma<1} L^1(\mathbb{R}, (1 + |x|^\gamma)dx)$ such that the operator A is J-nonnegative with the singular critical point zero and hence is not similar to a self-adjoint one.