Moment Problems for Real Measures on the Unit Circle

E. Lopushanskaya joint work with M. Bakonyi

In this talk we are considering the following problem: when are the given complex numbers $(c_j)_{j=-n}^n$, $c_{-j} = \bar{c_j}$, the first moments of a real Borel measure $\mu = \mu^+ - \mu^-$ on \mathbb{T} , such that μ^- is supported on a set of at most k points. A necessary and sufficient condition is that the Toeplitz matrix $T = (c_{i-j})_{i,j=0}^n$ is a certain real linear combination of rank 1 Toeplitz matrices. For k > 0, this is more general than the condition that T admits self-adjoint Toeplitz extensions with k negative squares. For a singular T, an equivalent condition is that a certain polynomial has all its roots on \mathbb{T} . We also discuss the situation when T is invertible.

The research is supported by the Russian Foundation for Basic Research, grant RFBR 05-01-00203-a