On the Riesz Basis Property in Elliptic Eigenvalue Problems with an Indefinite Weight Function

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We consider elliptic eigenvalue problems with indefinite weight function of the form

$$Lu = \lambda Bu, \quad x \in G \subset R^n, \tag{1}$$

$$B_j u|_\Gamma = 0, \quad j = \overline{1, m}, \tag{2}$$

where L is an elliptic differential operator of order $2m$ defined in a domain $G \subset R^n$ with boundary Γ, the B_j's are differential operators defined on Γ, and $Bu = g(x)u$ with $g(x)$ a measurable function changing a sign in G. We assume that there exist open subsets G^+ and G^- of G such that $\mu(\overline{G^\pm} \setminus G^\pm) = 0$ (μ is the Lebesgue measure), $g(x) > 0$ almost everywhere in G^+, $g(x) < 0$ almost everywhere in G^-, and $g(x) = 0$ almost everywhere in $G^0 = G \setminus (\overline{G^+} \cup \overline{G^-})$. For example, it is possible that $G^0 = \emptyset$. Let the symbol $L_{2,g}(G \setminus G^0)$ stand for the space of functions $u(x)$ measurable in $G^+ \cup G^-$ and such that $u|g|^{1/2} \in L_2(G \setminus G^0)$. Define also the spaces $L_{2,g}(G^+)$ and $L_{2,g}(G^-)$ by analogy.

We study the problems on the Riesz basis property of eigenfunctions and associated functions of problem (1)-(2) in the weighted space $L_{2,g}(G \setminus G^0)$ and the question on unconditional basisness of "halves" of eigenfunctions and associated functions in $L_{2,g}(G^+)$ and $L_{2,g}(G^-)$, respectively. If $L > 0$ then these halves comprise eigenfunctions corresponding to positive and negative eigenvalues. The latter problem is closely related to the former. Our approach is based on the interpolation theory for weighted Sobolev spaces. We refine known results. Our conditions on the weight g are connected with some integral inequalities.