Passive Impedance Bi-Stable Systems with Losses of Scattering Channels

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The conservative and passive impedance linear time invariant systems $\Sigma = (A, B, C,\ D; X, U)$ with discrete time and with Hilbert state and external spaces X and U respectively, and their impedances $c(z) = D+zC(I-zA)^{-1}B$ were studied earlier by different authors, see e.g.[1]. In our recent works [3]-[5] we concentrate our attention on the losses case. By this we understand the case, when the factorization inequalities

$$\varphi(z)^*\varphi(z) \le 2\Re c(z), \qquad \psi(z)\psi(z)^* \le 2\Re c(z), \quad z \in D, \tag{1}$$

have at least one nonzero solution $\varphi(z)$ and $\psi(z)$ in the classes of holomorphic inside open unite disk D functions with values from $\mathbb{B}(U, Y_\varphi)$ and $\mathbb{B}(U_\psi, U)$, respectively. Moreover, main results are relate to the bi-stable systems , i.e. to such systems, in which main operator A is a contraction from the class C_{00} that means

$$A^n \to 0 \quad \text{and} \quad (A^*)^n \to 0 \quad \text{when } n \to \infty.$$

To the impedance system Σ of such type corresponds the passive impedance systems with losses for which even factorizations equation

$$\varphi(\zeta)^*\varphi(\zeta) = 2\Re c(\zeta), \qquad \psi(\zeta)\psi(\zeta)^* = 2\Re c(\zeta), \quad \text{a. e. } |\zeta| = 1,$$

have nonzero solutions, which understands in weak sense for operator-valued functions. Such a system with impedance matrix $c(z)$ can be realized as a part of scattering-impedance lossless transmission minimal system $\tilde{\Sigma} = (\tilde{A}, \tilde{B}, \tilde{C}, \tilde{D}; \tilde{X}, \tilde{U}, \tilde{Y})$ with $\tilde{U} = U_1 \oplus U \oplus U_2$, $\tilde{Y} = Y_1 \oplus U \oplus Y_2$, where $U_2 = Y_2 = U$, by setting

$$X = \tilde{X}, \qquad A = \tilde{A}, \qquad B = \tilde{B}|U, \qquad C = P_U\tilde{C} \qquad \text{and} \qquad D = P_U\tilde{D}|U.$$

The system $\tilde{\Sigma}$ has the system operator

$$M_{\tilde{\Sigma}} = \begin{bmatrix} A & K & B & 0 \\ M & S & N & 0 \\ C & L & D & I_U \\ 0 & 0 & I_U & 0 \end{bmatrix}$$

that is $(\tilde{J}_1, \tilde{J}_2)$-unitary; transfer function $\tilde{\theta}_{J_1,J_2}(z)$ of system $\tilde{\Sigma}$ when $z \in D$ is (J_1, J_2)-bi-inner (in a certain weak sense) and has special structure

$$\tilde{\theta}_{J_1,J_2}(z) = \begin{bmatrix} \alpha(z) & \beta(z) & 0 \\ \gamma(z) & \delta(z) & I_U \\ 0 & I_U & 0 \end{bmatrix}, \qquad z \in D,$$

with 22-block $\delta(z)$ that equal to the impedance matrix $c(z)$ that belongs to the Caratheodory class, where

$$J_1 = \begin{bmatrix} I_{U_1} & 0 \\ 0 & J_U \end{bmatrix}, J_2 = \begin{bmatrix} I_{Y_1} & 0 \\ 0 & J_U \end{bmatrix}, J_U = \begin{bmatrix} 0 & -I_U \\ -I_U & 0 \end{bmatrix}, \tilde{J}_j = \begin{bmatrix} I_X & 0 \\ 0 & J_j \end{bmatrix},$$

$j = 1, 2$. If the main operator A of the system Σ belongs to the class C_0 in Nagy-Foias sense, then function $\tilde{\theta}_{J_1,J_2}(z)$ is meromorphic in the exterior D_e of disk D with bounded Nevanlinna characteristic in D_e. Moreover, meromorphic pseudocontinuation in D_e in weak sense of the restriction of $\tilde{\theta}_{J_1,J_2}(z)$ on D equals to $\tilde{\theta}_{J_1,J_2}(z)$ in D such that for any $\tilde{u} \in \tilde{U}, \tilde{y} \in \tilde{Y}$

$$\lim_{r\downarrow 1}(\tilde{\theta}_{J_1,J_2}(r\zeta)\tilde{u}, \tilde{y}) = \lim_{r\uparrow 1}(\tilde{\theta}_{J_1,J_2}(r\zeta)\tilde{u}, \tilde{y}) \qquad \text{a.e. } |\zeta| = 1.$$

Impedance matrix $c(z)$ of system Σ as a block of $\tilde{\theta}_{J_1,J_2}(z)$ is meromorphic in D_e with bounded Nevanlinna characteristic in D_e and for any $u_1, u_2 \in U$

$$\lim_{r\downarrow 1}(c(r\zeta)u_1, u_2) = \lim_{r\uparrow 1}(c(r\zeta)u_1, u_2) \qquad \text{a.e. } |\zeta| = 1.$$

Our results are intimately connected with work [2] where problems related to Surface Acoustic Wave filters are studied. In corresponding systems inputs are incoming waves and voltages and outputs are outgoing waves and currents, and transfer function is "mixing matrix"

$$\begin{bmatrix} \alpha & \beta \\ \gamma & \delta \end{bmatrix}$$

that is the main part of transmission matrix $\tilde{\theta}_{J_1,J_2}$ of system $\tilde{\Sigma}$ in our considerations.

In the case $\dim U < \infty$ the analytical problem of the description of the set of corresponding lossless scattering-impedance transmission matrices with given 22-block $\delta = c$ was studied in [3]. The present here results can be found in [4], [5].

References

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