Asymptotics of Eigenvalues of a Sturm–Liouville Problem with Discrete Self-Similar Indefinite Weight

I. A. Sheipak
joint work with A. A. Vladimirov

We study the asymptotics of the spectrum for the boundary eigenvalue problem

$$-y'' - \lambda\rho y = 0, \tag{1}$$
$$y(0) = y(1) = 0, \tag{2}$$

where $\rho \in \overset{\circ}{W}{}_2^{-1}[0,1]$ is the generalized derivative of fractal (self-similar) piecewise function $P \in L_2[0,1]$.

The numbers $n \in \mathbb{N}$, $n \geqslant 2$, q ($|q| > 1$), $Z_+, Z_- \geqslant 0$ can be defined via parameters of self-similarity of function P.

Our main results are the following:

1. If $q > 1$, $Z_+ > 0$ and $Z_+ + Z_- = n - 1$ then there are numbers $\mu_l > 0$, $l = 1, 2, \ldots, Z_+$, such that positive eigenvalues $\{\lambda_k\}_{k=1}^{\infty}$ of the problem (1)–(2) have the asymptotics
$$\lambda_{l+kZ_+} = \mu_l \cdot q^k (1 + o(1)).$$

2. If $q > 1$, $Z_- > 0$ and $Z_+ + Z_- = n - 1$ then there are numbers $\mu_l > 0$, $l = 1, 2, \ldots, Z_-$, such that negative eigenvalues $\{\lambda_{-k}\}_{k=1}^{\infty}$ of the problem (1)–(2) have the asymptotics
$$\lambda_{-(l+kZ_-)} = -\mu_l \cdot q^k (1 + o(1)).$$

3. If $q < -1$, $Z_+ + Z_- = n - 1$ then there are numbers $\mu_l > 0$, $l = 1, 2, \ldots, n-1$, such that that positive eigenvalues $\{\lambda_k\}_{k=1}^{\infty}$ of the problem (1)–(2) have the asymptotics

$$\lambda_{l+k(n-1)} = \mu_l \cdot |q|^{2k}(1 + o(1))$$

and negative eigenvalues $\{\lambda_{-k}\}_{k=1}^{\infty}$ of the problem (1)–(2) have the asymptotics

$$\lambda_{-(l+Z_-+k(n-1))} = -\mu_l \cdot |q|^{2k+1}(1 + o(1)).$$

All these results are new even if function ρ is positive.

Asymptotics of eigenvalues of Sturm–Liouville problem with discrete self-similar weight//(http://arxiv.org/abs/0709.0424)