Commuting Domination in Pontryagin Spaces

M. Wojtylak

We will discuss the following theorem, proved originally in [2] for formally normal and normal operators in Hilbert spaces.

Theorem 1 *Let $A_0, \dots, A_n$ $(n \geq 1)$ be symmetric operators in a Pontryagin space $\mathcal{K}$ and let $\mathcal{E}_{ij}$, $0 \leq i < j \leq n$, be dense linear spaces of $\mathcal{K}$ such that*

(i) *A_j weakly commutes with A_0 on $\mathcal{E}_{0j}$ for $j = 1, \dots, n$;*

(ii) *A_i pointwise commutes with A_j on $\mathcal{E}_{ij}$ for $1 \leq i < j \leq n$;*

(iii) *A_0 is essentially selfajoint on $\mathcal{E}_{0j}$ for $j = 1, \dots, n$;*

(iv) *A_0 dominates A_j on $\mathcal{E}_{0j}$ for $j = 1, \dots, n$.*

Then $\bar{A}_0, \dots, \bar{A}_n$ are spectrally commuting selfadjoint operators.

The proof requires some results on bouded vectors of a selfadjoint operator in a Pontryagin space. As a corollary we obtain a polynomial version of Nelson's criteria for selfadjointness. We will use the theory of operator matrices with all unbounded entries, which was developed in [1].

References

[1] M. Möller, F.H. Szafraniec, Adjoints and formal adjoints of matrices of unbounded operators, *Proc. Amer. Math. Soc.*

[2] J. Stochel, F. H. Szafraniec, Domination of unbounded operators and commutativity, *J. Math. Soc. Japan,* 55 No.2, (2003), 405-437.

[3] M. Wojtylak, Commuting domination in Pontryagin spaces, preprint