Friday, December 17th

9:00 – 9:10   *Opening*

Chair: Vadim Adamyan

9:10 – 9:40   **Heinz Langer**
Self-adjoint analytic operator functions

9:40 – 10:10   **Henk de Snoo**
A canonical decomposition for linear operators
and linear relations

10:10 – 10:40   **Vladimir Derkach**
Asymptotic expansions of generalized Nevanlinna
functions

10:40 – 11:10   **Andras Batkai**
Differential operators with Wentzell boundary
conditions: An operator matrix approach

11:10 – 12:00   *Refund of travel expenses (MA 674)*

                & *Coffee break (MA 464)*

Friday, December 17th

Chair: Branko Ćurgus

12:00 – 12:30   **Leiba Rodman**
Extension of jet functions with indefinite Cara-
theodory-Pick Matrices

12:30 – 13:00   **Annemarie Luger**
On the eigenvalues of an abstract λ-dependent
boundary value problem

13:00 – 13:30   **Andrei Shkalikov**
Invariant subspaces of dissipative operators in
Krein spaces

13:30 – 15:00   *Lunch break*

Chair: Andreas Lasarow

Chair: Heinz Langer

15:00 – 15:30 **Tomas Azizov**
On the number of fixed points of holomorphisms

15:30 – 16:00 **Seppo Hassi**
A class of Nevanlinna functions related to singular Sturm-Liouville problems

16:00 – 16:30 **Franciszek Szafraniec**
Favards's theorem modulo an ideal

16:30 – 17:00 **Adrian Sandovici**
Degenerate inner product spaces

17:00 – 17:30 *Coffee break (DFG Lounge MA 315)*

17:30 – 18:00 **Yury Arlinskii**
Passive quasi-selfadjoint discrete-time systems

18:00 – 18:30 **Henrik Winkler**
Extremal extensions of nonnegative operators

18:30 – 19:00 **Birgit Jacob**
On null-controllability of diagonal systems

19:00 – 19:30 **Jussi Behrndt**
Finite rank perturbations of locally definitizable selfadjoint operators

20:30 *Conference dinner*

ZIKO'S GRILL, Kaiser-Friedrich-Str. 61a

Saturday, December 18th

Chair: Björn Textorius

9:00 – 9:30   **Aad Dijksma**
Rank one perturbations at infinite coupling
in Pontryagin spaces

9:30 – 10:00   **Christiane Tretter**
A Krein space approach to the Klein-Gordon
equation

10:00 – 10:30   **Volker Mehrmann**
Quadratic operator eigenvalue problems with
hamiltonian or symplectic eigensymmetry

10:30 – 11:00   **Andre Ran**
Integral operators with semi-separable kernels
with symmetries

11:00 – 11:30   *Conference photo*

       &   *Coffee break (DFG Lounge MA 315)*

Chair: Matthias Langer

11:30 – 12:00   **Kresimir Veselic**
Some new results on damped systems
(Perturbation bounds and exponential decay)

12:00 – 12:30   **Branko Curgus**
Riesz basis of root vectors of indefinite Sturm-
Liouville problems

12:30 – 13:00   **Andreas Fleige**
The green-red colouring of Curgus for indefinite
Sturm-Liouville operators with certain interface
conditions

13:00 – 14:30   *Lunch break*

Saturday, December 18th

Chair: Henk de Snoo

Chair: Michael Kaltenbäck

14:30 – 15:00   **Daniel Alpay**
Schur-Takagi Problem and generalized Schur
functions: The time-varying case

15:00 – 15:30   **Michael Kaltenbäck**
Symmetric relations of finite negativity

15:30 – 16:00   **Harald Woracek**
De Branges spaces of entire functions symmetric
about the origin

16:00 – 16:30   **Rostyslav Hryniv**
Inverse problems for compound oscillating systems

16:30 – 17:00   *Coffee Break (DFG Lounge MA 315)*

17:00 – 17:30   **Aurelian Gheondea**
Lifting of operators to Hilbert spaces induced
by positive selfadjoint operators

17:30 – 18:00   **Gerald Wanjala**
Generalized Schur functions and augmented
Schur parameters

18:00 – 18:30   **Vyacheslav Pivovarchik**
Spectral properties of a fourth order differential
equation

18:30 – 19:00   **Victor Khatskevich**
Operator pencils of the second order and lin-
ear fractional relations

Chair: Henrik Winkler

Chair: Christiane Tretter

9:00 – 9:30   **Vadim Adamyan**
Entire matrix functions orthogonal with respect to some non-positive matrix measures on the real axis

9:30 – 10:00   **Matthias Langer**
Resonances and spectral concentration of a Sturm-Liouville problem depending rationally on the eigenvalue parameter

10:00 – 10:30   **Mark Malamud**
Borg type results for matrix Sturm-Liouville operator

10:30 – 11:00   **Vladimir Strauss**
On the uniqueness of spectral functions for WJ^*-algebras of D_κ^+-class

11:00 – 11:30   *Coffee break (DFG Lounge MA 315)*

11:30 – 12:00   **Yuri Shondin**
Approximation of high order singular perturbations

12:00 – 12:30   **Carsten Trunk**
Uniformly dissipative perturbations of selfadjoint operators in Krein spaces

12:30 – 13:00   **Monika Winklmeier**
A variational principle for unbounded block operator matrices and applications

13:00 – 14:00   *Lunch break*

Sunday, December 19th

Chair: Aad Dijksma

14:00 – 14:30   **Hagen Neidhardt**
Zeno product formula

14:30 – 15:00   **Ilia Karabash**
Similarity of J-selfadjoint differential operators
to selfadjoint ones

15:00 – 15:30   **Lyudmila Sukhocheva**
On selfadjoint operators in degenerate Krein
spaces

15:30 – 16:00   **Peter Jonas**
On operator representations of locally defini-
tizable operator functions

16:00 - 16:10   *Closing*

Entire matrix functions orthogonal with respect to some non-positive matrix measures on the real axis

Vadim Adamyan

Let $\omega(\lambda)$, $-\infty < \lambda < \infty$, be a real symmetric $n \times n$ matrix function of limited variation. We assume that the matrix function

$$\sigma(\lambda) := \frac{\lambda}{2\pi} I + \omega(\lambda),$$

with the $n \times n$ unity matrix I, admits the representation $\sigma(\lambda) \equiv \sigma_+(\lambda) - \sigma_-(\lambda)$, where $\sigma_{\pm}(\lambda)$ are non-decreasing matrix functions, σ_- has at most $\varkappa < \infty$ growing-points. In this talk we introduce following M.G. Krein the system of entire matrix functions which are orthogonal with respect to the measure $d\sigma(\lambda)$ and discuss some of their properties.

Schur–Takagi problem and generalized Schur functions: The time–varying case

Daniel Alpay

The Carathéodory–Fejér extension problem for functions analytic and contractive in the open unit disk $\mathbb{D}$ (Schur functions) was studied by Takagi in 1924 when the analyticity requirement is removed and functions are allowed to have poles in $\mathbb{D}$.

In this talk we consider the Schur–Takagi problem in the time–varying case, that is when one replace Schur functions by upper triangular contractions, the complex variable by a bilateral backward shift Z and the complex numbers by diagonal operators. An important tool is the Zadeh transform associated to an upper triangular operator $U = \sum_{n=0}^{\infty} Z^n U_{[n]}$, and defined by

$$U(\rho) = \sum_{n=0}^{\infty} \rho^n Z^n U_{[n]}.$$

We adapt the reproducing kernel approach to the generalized Schur algorithm presented in [1] to the time–varying case. Now Kreĭn spaces rather than Pontryagin spaces come into play.

This is joint work with Patrick Dewilde and Dan Volok (Delft University of Technology).

[1] D. Alpay, T. Azizov, A. Dijksma, and H. Langer. The Schur algorithm for generalized Schur functions. III. J-unitary matrix polynomials on the circle. *Linear Algebra Appl.*, 369:113–144, 2003.

Passive quasi-selfadjoint discrete-time systems

Yury Arlinskii

The discrete-time system

$$\tau = \left\{ \begin{pmatrix} A & B \\ C & D \end{pmatrix}, H, \mathfrak{N}, \mathfrak{N} \right\}$$

is called passive quasi-selfadjoint if the operator $T = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$ is a contraction in the Hilbert space $\mathcal{H} = H \oplus \mathfrak{N}$ and $\operatorname{ran}(T - T^*) \subset \mathfrak{N}$.

The transfer function $\Theta(z) = D + zC(I - zA)^{-1}B$ of this system belongs to a subclass $\mathbf{S}^{qs}(\mathfrak{N})$ of the Schur class $\mathbf{S}(\mathfrak{N})$. We give characterizations of the functions belonging to the subclass $\mathbf{S}^{qs}(\mathfrak{N})$ and study the properties of their minimal realizations.

The talk is based on the joint work with S. Hassi (Vaasa University) and H.S. de Snoo (Groningen University).

On the number of fixed points of holomorphisms

Tomas Azizov

Under some conditions a family of holomorphisms has a unique fixed point or the set of its fixed points is affine.

This talk is based on a joint paper with V. Khatskevich and D. Shoikhet (Israel).

Differential operators with Wentzell boundary conditions: An operator matrix approach

Andras Batkai

Second order ordinary differential operators with Wentzell boundary conditions will be considered. Using similarity transformations, the operator will be transformed into a matrix operator on a product space. This method will be used to investigate functional anaytic properties of the differential operator. Special emphasis will be laid on the generation of cosine families.

[1] Bátkai, A., Engel, K.-J., *Abstract wave equations with generalized Wentzell boundary conditions*, J. Diff. Eqs. **207** (2004), 1 – 20.

[2] Bátkai, A., Engel, K.-J., Haase, M., *Cosine families generated by second order differential operators on $W^{1,1}(0,1)$ with generalized Wentzell boundary conditions*, submitted

Finite rank perturbations of locally definitizable selfadjoint operators

Jussi Behrndt

It was shown by P. Jonas and H. Langer that a definitizable operator remains definitizable after a finite dimensional perturbation in resolvent sense if the unperturbed and the perturbed operator have a common point in their resolvent sets.

We show that a similar statement holds true for selfadjoint operators which have only locally the same spectral properties as definitizable operators.

[1] P. Jonas, H. Langer: Compact Perturbations of Definitizable Operators, J. Operator Theory **2** (1979), 63-77.

Riesz basis of root vectors of indefinite Sturm-Liouville problems

Branko Ćurgus

I shall consider a regular indefinite Sturm-Liouville eigenvalue problem

$$-(pf')' + qf = \lambda\, r f \quad \text{on} \quad [-1, 1]$$

with self-adjoint boundary conditions. The coefficients $1/p$, q, r, are real integrable functions on $[-1, 1]$, $p(x) > 0$ and $x\, r(x) > 0$ for almost all $x \in [-1, 1]$. This eigenvalue problem determines a definitizable operator A in the Krein space $L_{2,r}(-1, 1)$. The operator A has discrete spectrum. I shall consider basis properties of root vectors of A in two spaces: the form domain of A and $L_{2,r}(-1, 1)$.

There exists a Riesz basis of the form domain of A consisting of the root vectors of A. This property holds without any additional conditions on the coefficients p, q and r.

The situation in $L_{2,r}(-1, 1)$ is more complicated. Recently A. I. Parfënov considered $p = 1, q = 0$, an odd weight function r and the Dirichlet boundary conditions. Parfënov gave a necessary and sufficient condition for r under which there exists a Riesz basis of $L_{2,r}(-1, 1)$ consisting of eigenvectors of A. Only sufficient conditions for the existence of Riesz basis of $L_{2,r}$ consisting of root vectors of A are available for more general boundary conditions and functions r which are not odd. I shall discuss such conditions for r and how they relate to the boundary conditions.

Asymptotic expansions of generalized Nevanlinna functions

Vladimir Derkach

The generalized Nevanlinna class $\mathbf{N}_{\kappa,0}$ ($\kappa \in \mathbb{Z}_+$) consists of generalized Nevanlinna functions $Q \in \mathbf{N}_\kappa$ which satisfy the condition $Q(z) = \gamma + O(1/z)$, $z \hat{\to} \infty$ with a real γ. As is known Q belongs to the class $\mathbf{N}_{\kappa,0}$ if and only if it admits the operator representation

$$Q(z) = \gamma + [(A - z)^{-1}\omega, \omega]$$

with A being a selfadjoint operator in a Pontryagin space $(\mathfrak{H}, [\cdot, \cdot])$ and $\omega \in \mathfrak{H}$. Let S be the restriction of A to

$$\operatorname{dom} S = \{\, f \in \operatorname{dom} A : \ [f, \omega] = 0 \,\}.$$

We characterize the asymptotic expansions of Q at ∞ both in terms of its operator and integral representations and in terms of the structure of the root subspace of the generalized Friedrichs extension S_F of the symmetric operator S.

This is joint work with Seppo Hassi (University of Vaasa) and Henk de Snoo (University of Groningen).

Rank one perturbations at infinite coupling in Pontryagin spaces

Aad Dijksma

In this lecture we report on joint work with Heinz Langer and Yuri Shondin. We relate the operators in the operator representations of a generalized Nevanlinna function $N(z)$ and of the function $-N(z)^{-1}$ under the assumption that $z = \infty$ is the only (generalized) pole of non-positive type. The results are applied to the Q-function of S and H and the Q-function for S and H^∞, where H is a self-adjoint operator in a Pontryagin space with a cyclic element w, H^∞ is the self-adjoint relation obtained from H and w via a rank one perturbation at infinite coupling, and S is the symmetric operator given by $S = H \cap H^\infty$.

The green-red colouring of Ćurgus for indefinite Sturm-Liouville operators with certain interface conditions

Andreas Fleige

We consider the indefinite Sturm-Liouville eigenvalue equation

$$-f'' = \lambda r f$$

where the weight function r is negative on $[-1, 0)$ and positive on $(0, 1]$. Last year in Berlin Branko Ćurgus parametrized the boundary conditions of all selfadjoint extensions of the associated minimal symmetric operator in the Krein space $L_r^2([-1, 1])$ by parameters describing a sphere. Here we fix one of these boundary conditions but impose an additional interface condition at the so called "turning point" $x = 0$ of r. Then all selfadjoint (and definitizable) extensions of the corresponding symmetric operator can be parametrized by a circle instead of a sphere. According to Ćurgus' approach we colour a point of the circle red (green) if infinity is (not) a singular critical point of the corresponding definitizable operator. The essential result is the fact that regardless of the behaviour of r the circle is totally green with the exception of one point which can also be red.

Lifting of operators to Hilbert spaces induced by positive selfadjoint operators

Aurelian Gheondea

We introduce the notion of induced Hilbert spaces for positive unbounded operators and show that the energy spaces associated to several classical boundary value problems for partial differential equations are relevant examples of this type. The main result is a generalization of the Krein-Reid lifting theorem to this unbounded case and we indicate how it provides estimates of the spectra of operators with respect to energy spaces.

This is based on joint work with P. Cojuhari (Cracow).

A class of Nevanlinna functions related to singular Sturm-Liouville problems

Seppo Hassi

The class of Nevanlinna functions consists of functions which are holomorphic off the real axis, which are symmetric with respect to the real axis, and whose imaginary part is nonnegative in the upper halfplane. The so-called Kac subclass of Nevanlinna functions is defined by an integrability condition on the imaginary part. In this talk a further subclass of the Kac functions is considered. It involves an integrability condition on the modulus of Nevanlinna functions (instead of the imaginary part). The characteristic properties of functions in this subclass are treated. The definition of this subclass is motivated by the fact that the Titchmarsh-Weyl coefficients of various classes of Sturm-Liouville problems under mild conditions on the coefficients actually belong to this subclass.

The talk is based on joint work with Manfred Möller and Henk de Snoo.

Inverse problems for compound oscillating systems

Rostyslav Hryniv

We study the inverse problem of reconstruction of an oscillating system composed of a singular string and a spring with n adjoined masses from the eigenfrequences and the norming constants. The corresponding operator has a 2×2 block structure in $L_2(0,1) \times \mathbb{C}^n$, with a Sturm–Liouville operator and a Jacobi matrix on the diagonal and an off-diagonal coupling, and is selfadjoint in the corresponding Hilbert or Pontryagin space. The eigenvalue problem is reduced to that for a Sturm–Liouville equation subject to the boundary conditions depending rationally on the spectral parameter. We give a complete solution to the direct and inverse spectral problem, i.e., find necessary and sufficient conditions for the spectral data and provide the reconstruction algorithm.

On null-controllability of diagonal systems

Birgit Jacob

In this talk we develop necessary and sufficient conditions for null-controllability of diagonal systems with a one-dimensional input operator. We do not assume that the input operator is bounded or admissible; thus all boundary and point controls are included. The necessary and sufficient conditions are in terms of the eigenvalues of the systems operator and of the coeffients of the input operator. Further, we give conditions for exact controllability as well.

On operator representations of locally definitizable operator functions

Peter Jonas

In this talk we consider locally definitizable operator functions and their representations with the help of selfadjoint operators and relations in Krein spaces. It is shown that there exist minimal representations such that the spectrum of the representing relation "locally coincides" with the set of points of nonholomorphy of the operator function.

Symmetric relations of finite negativity

Michael Kaltenbäck

We construct and investigate a space which is related to a symmetric linear relation S of finite negativity on an almost Pontryagin space. This space is the indefinite generalization of the completion of $\operatorname{dom} S$ with respect to $(S., .)$ for a strictly positive S on a Hilbert space.

Similarity of J-selfadjoint differential operators to selfadjoint ones

Ilia Karabash

Let L be a selfadjoint differential operator, J is the multiplication operator by sgn x. The question is the similarity of J-selfadjoint operator $A = JL$ to a selfadjoint operator.

The first approach is based on Krein-Langer spectral theory of definitizable operators in a Krein space. If $L > \delta > 0$ the results of B. Ćurgus, H. Langer, B. Najman, M. Faierman on the regularity of the critical point ∞ imply that JL is similar to selfadjoint operator for a wide class of differential operators L. If $\inf \sigma_c(L) = 0$ the question is more difficult, since it leads to the investigation of the regularity of critical point 0. What is why only several model operators were studied in this case.

The aim of the talk is to apply Naboko-Malamud similarity criterion to J-selfadjoint differential operators. This method gives new results if $\inf \sigma_c(L) = 0$ and allows to deal with the case $\sigma_c(L) \cap (-\infty, 0) \neq \emptyset$.

1. Let $L = p(-id/dx)$, where $p(t) = t^{2n} + a_{2n-1}t^{2n-1} + \cdots + a_0$ is a polynomial with real coefficients. Then the operator

$$A = (\operatorname{sgn} x)p(-id/dx)$$

acting in $L^2(\mathbb{R})$ is similar to a selfadjoint operator iff p is nonnegative [1].

2. In [2] we consider $L = -d^2/dx^2 + q(x)$ in $L^2(\mathbb{R})$, with a finite-zone potential q. In this case $\sigma_{disc}(A)$ is finite and A admits a decomposition $A = A_{disc} \dot{+} A_{ess}$, $\sigma(A_{disc}) = \sigma_{disc}(A)$, $\sigma(A_{ess}) = \sigma_{ess}(A) \subset \mathbb{R}$. The criterion of similarity to selfadjoint for A_{ess} is obtained. The example of operator JL which is similar to selfadjoint and $\sigma_{ac}(L) \cap (-\infty, 0) \neq \emptyset$ is given.

3. Let Q be a lower semibounded selfadjoint operator acting in a separable Hilbert space $\mathcal{H}$. Consider a J-selfadjoint Sturm-Liouville operator with an operator potential

$$A = (\operatorname{sgn} x)(-d^2/dx^2 + Q)$$

in the space of vector-functions $L^2(\mathbb{R}, \mathcal{H})$. Then the spectrum of A is real. The operator A is similar to a selfadjoint operator iff the operator Q is nonnegative [3]. Different applications of this result to J-selfadjoint partial differential operators are given.

[1] I.M. Karabash, *J-selfadjoint ordinary differential operators similar to selfadjoint operators* // Methods Funct. Anal. Topology, 2000, V. 6, no. 2., P.22–49.

[2] I.M. Karabash, M.M. Malamud, *On similarity of J-selfadjoint Sturm-Liouville operators with finite-gap potential to selfadjoint ones* // Dokl. RAN., 2004, V. 394, no. 4., P.17–21. (Russian)

[3] S. Hassi, I.M. Karabash, *Similarity of J-selfadjoint Sturm-Liuoville operators with an operator potential to selfadjoint ones* // Spectral and Evolution problems, Proc. of the Fourteenth Crimean Autumn Math. School-Symposium, Simferopol, 2004, V. 14, P. 44-46.

Joint work with: Part 2. Mark Malamud (Donetsk National University), Part 3. Seppo Hassi (University of Vaasa).

Operator pencils of the second order and linear fractional relations

Victor Khatskevich

Two relatively new notions, those of operator pencils of the second order $P(X, Y)$:

$$P(X, Y) = A + XB + CY + XDY,$$

where A, B, C, D are given operators in Banach spaces, and X, Y are operator variables, and of operator linear fractional relations (LFR) F:

$$F(X) = \{X' : A + BX = X'(C + DX)\}$$

are considered. We study these notions in their interaction, somewhat arbitrarily considering LFR's as object of study and operator pencils $P(X, Y)$ as the method of their study. In reality the interaction of these notions is more complex. We consider the following three particular cases:

1. Linear pencils and general properties of LFR.

2. Quadratic pencils and fixed points of LFR.

3. Selfadjoint quadratic pencils and geometrical and topological structure of the image and the preimage of LFR.

In addition we study numerous applied problems related to these notions.

Self-adjoint analytic operator functions

Heinz Langer

For a self-adjoint analytic operator function, which satisfies the Virozub-Matsaev condition on some interval, a linearization in a Krein space and a local spectral function are defined and studied. Joint work with A.Markus and V.Matsaev.

Resonances and spectral concentration of a Sturm–Liouville problem depending rationally on the eigenvalue parameter

Matthias Langer

We consider resonances (i.e. non-real poles of the analytic continuation of the resolvent of an associated operator across the essential spectrum) of the following Sturm–Liouville problem

$$-y'' + \left(q - \lambda - t\frac{w}{u - \lambda}\right)y = 0$$

on the interval $[0, 1]$ which depends rationally on the eigenvalue parameter λ subject to Dirichlet boundary conditions. Here q, w and u are real analytic functions, and t is a coupling constant. The behaviour of resonances in dependence on t is investigated and the connection to spectral concentrations is shown. In particular the behaviour of the resonances as t tends to 0 is considered. Moreover, numerical results for some examples are presented.

On the eigenvalues of an abstract λ-dependent boundary value problem

Annemarie Luger

Let $A \subset A^+$ be a closed symmetric relation in a Krein space $\mathcal{H}$ with defect n and assume that there exists a self adjoint extension A_0 in $\mathcal{H}$ which is of type π_+ over some symmetric domain $\Omega \subset \overline{\mathbb{C}}$. Let the triple $(\mathbb{C}^n, \Gamma_0, \Gamma_1)$ with $A_0 = \ker \Gamma_0$ be a boundary value space for A^+ and denote the corresponding Weyl function by $M(\lambda)$.

In this situation we are considering the following eigenvalue dependent boundary value problem:

For a given $h \in \mathcal{H}$ find a vector $\hat{f} = \begin{pmatrix} f \\ f' \end{pmatrix} \in A^+$ such that

$$f' - \lambda f = h \qquad \text{with} \qquad \tau(\lambda)\Gamma_0 \hat{f} + \Gamma_1 \hat{f} = 0$$

holds. Here τ is a local generalized Nevanlinna function.

For λ in the domain of holomorphy of τ the linearization $\widetilde{A}$ of the problem provides all the information about its solvability. In this talk we show that, in particular, the eigenvalues and their type can also be expressed in terms of analytic properties of the function $M(\lambda) + \tau(\lambda)$. Moreover, we are discussing for which points $\lambda \notin \mathrm{hol}(\tau)$ the problem can still be described by the linearization $\widetilde{A}$.

This is joint work with Jussi Behrndt.

Borg type results for matrix Sturm-Liouville operator

Mark Malamud

The problem of the unique determination of the matrix Sturm-Liouville operator in $L^2[0, 1]$ with a summable potential matrix Q, not necessarily selfadjoint, given a part of the monodromy matrix, will be discussed. We present generalizations of two classical results on the unique determination of the scalar Sturm-Liouville operator by two spectra (the Borg result) and by one spectrum and half of a potential (the Hochstadt-Lieberman result) to the case of the matrix Sturm-Liouville equation.

Quadratic operator eigenvalue problems with hamiltonian or symplectic eigensymmetry

Volker Mehrmann

Many important applications lead to quadratic operator eigenvalue problems of the form

$$\lambda^2 m(u, v) + \lambda g(u, v) + k(u, v) = 0$$

with sesquilinear forms m, g, k.

We discuss a problem of computing singularity exponents in elastic field computations. We show how finite element discretizations lead to eigenvalue problems with specific structure and also briefly discuss numerical solution methods.

This is joint work with Thomas Apel and David Watkins

Zeno product formula

Hagen Neidhardt

We demonstrate a pair of new product formulæ which combine a projection with a resolvent of a positive operator, or with an exponential function and spectral projections, respectively. The convergence is strong or even operator-norm under more restrictive assumptions. The second mentioned formula can be used to describe Zeno dynamics in the situation when the usual non-decay measurement is replaced by a particular generalized observable in the sense of Davies.

This is a joint work with P. Exner, T. Ichinose and V. A. Zagrebnov.

Spectral properties of a fourth order differential equation

Vyacheslav Pivovarchik

The eigenvalue problem

$$y^{(4)}(\lambda, x) - (gy')'(\lambda, x) = \lambda^2 y(\lambda, x)$$

$$y(\lambda, 0) = y''(\lambda, 0) = y(\lambda, a) = y''(\lambda, a) + i\lambda\alpha y'(\lambda, a) = 0$$

is considered, where $g \in C^1[0, a]$ is real valued and $\alpha > 0$. It is shown that the eigenvalues lie in the closed upper half-plane and on the negative imaginary half-axis. A formula for the asymptotic distribution of the eigenvalues is given and the location of the pure imaginary spectrum is investigated.

This is a joint work with Manfred Möller from Johannesburg.

Integral operators with semi-separable kernels with symmetries

André Ran

We characterize integral operators with semi-separable kernels in a certain class that have different symmetries. We treat the selfadjoint case and the J-unitary case, and comment on the positive case, the positive real case, the dissipative case and the contractive case.

This is joint work with G.J. Groenewald and M.A. Petersen, University of the North-West (Potchefstroom), South Africa.

Extension of jet functions with indefinite Carathéodory - Pick matrices

Leiba Rodman

The talk is based on joint work with Vladimir Bolotnikov and Alexander Kheifets.

A class of functions is introduced that take values in the set of ordered tuples of complex numbers and are defined on a subset of the unit disc. The class is defined by the property that all Carathéodory–Pick matrices of a function have not more than a prescribed number of negative eigenvalues, and at least one Carathéodory–Pick matrix of the function has exactly the prescribed number of negative eigenvalues. The class is characterized in several ways. It turns out that a typical function in the class is generated by a meromorphic function, together with several of its derivatives at regular points, with a possible modification at a finite number of points. Extension and interpolation results are proved for functions in the class.

Degenerate inner product spaces

Adrian Sandovici

Let S be a linear relation in a degenerated inner product space H. Then S can be viewed as a symmetric linear relation in H. All its selfadjoint extensions can be characterized in terms of von Neumann and Krein formulas. Some classes of matrix valued Nevanlinna functions are also involved. They are related by a decomposition of a linear relation in a Euclidian space.

The talk is based on joint work with Henk de Snoo and Henrik Winkler.

Invariant subspaces of dissipative operators in Krein spaces

Andrei Shkalikov

We study dissipative operators in Krein spaces $\mathcal{K} = \{\mathcal{H}, J\}$ with fundamental symmetry $J = P_+ - P_-$. All generalizations of Pontryagin theorem on the existence of invariant maximal definite subspaces for selfadjoint and dissipative operators assume the Langer condition $\mathcal{D}(A) \supset H_+ = P_+(H)$, where $\mathcal{D}(A)$ is the domain of an operator A. We prove the theorems on invariant subspaces starting from a weaker assumption. Namely, we assume only that the linear manifolds $\mathcal{D}(A) \cap H_\pm$ are dense in $H_\pm$.

Approximation of high order singular perturbations

Yuri Shondin

Let $\mathcal{H}$, $\langle \cdot, \cdot \rangle$ be a Hilbert space, L be a positive self-adjoint operator in $\mathcal{H}$ and φ be an element of the space $\mathcal{H}_{-k-1} \setminus \mathcal{H}_{-k}$, $k \geq 2$. Here $\mathcal{H}_l$'s are the Hilbert scale spaces with the inner products $\langle (L+1)^l \cdot, \cdot \rangle$.

For a given set $\widehat{g} = \{g_s\}_{s=2}^k$ of real numbers m-model (realization) for singular perturbation of L generated by φ is described by the triple $\Pi(\widehat{g}), S(\widehat{g}), H^g(\widehat{g})$, where $\Pi(\widehat{g})$ is a π_m–space with $m = [\frac{k}{2}]$, $S(\widehat{g})$ is a symmetric operator with defect indices $(1,1)$ in $\Pi(\widehat{g})$ and $H^g(\widehat{g})$, $g \in \mathbb{R} \cup \infty$, is one parameter family of canonical s.a. extensions of $S(\widehat{g})$. With $\mu < 0$ the function

$$Q_k(z) = \langle \frac{(z-\mu)^k}{(L-z)(L-\mu)^k} \varphi, \varphi \rangle_0 + \sum_{l=0}^{k-1} g_{j+1}(z-\mu)^l, \quad g_1 \equiv g,$$

belongs to the class $\mathcal{N}_m$ and it is Q-function for $S(\widehat{g})$ and $H^\infty(\widehat{g})$.

For an "approximant" we take a smother $(1k)$–model , which is determined by L, an element $\psi \in \mathcal{H}_{-2} \setminus \mathcal{H}_0$ and real parameters from $\widehat{\gamma} = \{\gamma_s\}_{s=2}^k$ and is described by the triple $\mathcal{K}(\widehat{\gamma}), S(\widehat{\gamma}), A^\gamma(\widehat{\gamma})$, where $\Pi(\widehat{g})$ is a π_κ–space, $S(\widehat{\gamma})$ is a symmetry with defect indices $(1,1)$ in $\Pi(\widehat{g})$ and $A^\gamma(\widehat{\gamma})$, $\gamma \in \mathbb{R} \cup \infty$, is one parameter family of canonical s.a. extensions of $S(\widehat{\gamma})$. The function

$$Q_{1k}(z) = \langle \frac{(z-\mu)}{(L-z)(L-\mu)} \psi, \psi \rangle_0 + \sum_{l=0}^{k-1} \gamma_{l+1}(z-\mu)^k, \quad \gamma_1 \equiv \gamma,$$

belongs to the class $\mathcal{N}_\kappa$ and it is Q-function for $S(\widehat{\gamma})$ and $A^\infty(\widehat{\gamma})$.

Let sequences of numbers $\gamma_s^{(n)}$, $s = \overline{2,k}$, and elements $\psi^{(n)}$ from $\mathcal{H}_{-2}$ are taken in such way that $\psi^{(n)} \xrightarrow{n \to \infty} \varphi$ in $\mathcal{H}_{-k-1}$ and

$$\gamma_s^{(n)} + \langle (L-\mu)^{-s} \psi^{(n)}, \psi^{(n)} \rangle_0 \xrightarrow{n \to \infty} g_s,$$

then the spaces $\mathcal{K}(\widehat{\gamma}^{(n)})$ strongly approximates the space $\Pi(\widehat{g})$ and for $z \in \rho(H^g(\hat{g}))$ the sequence $(A^g(\hat{\gamma}^{(n)}) - z)^{-1}$ strongly approximates the resolvent $(H^g(\widehat{q}) - z)^{-1}$ in the sense of approximation with variable spaces. An example of approximation by a boundary condition will be discussed.

A canonical decomposition for linear operators and linear relations

Henk de Snoo

Any linear relation is shown to be the sum of a closable operator and a singular relation, whose closure is the Cartesian product of closed subspaces. These two parts are characterized metrically and in terms of Stone's characteristic projection onto the closure of the linear relation.

This is joint work with S. Hassi, Z. Sebestyén, and F.H. Szafraniec.

On the uniqueness of spectral functions for WJ^*-algebras of D_κ^+-class

Vladimir Strauss

A goal of this report is a study of relations between commutative WJ^*-algebras in separable Krein spaces and some spectral function. It is assumed that a WJ^*-algebra has a maximal non-negative invariant subspace, presented as a direct sum of a neutral subspace with finite dimension and a uniformly positive subspace. As it is known this algebra generates a spectral function E_λ with a peculiar spectral set Λ that provides a resolution of spectral type for the operators from the algebra. In particular with every operator A one can associate a scalar function $f_A(\lambda)$ (the image of A) such that

$$AE(\Delta) = \int_\Delta f_A(\lambda)dE_\lambda,$$

where Δ runs the set of all closed intervals of the real line disjoint with Λ.

Note that for the given algebra its spectral function E_λ is not uniquely determined. We study the problem of arbitrariness for the choice of E_λ and some related topics.

On selfadjoint operators in degenerate Krein spaces

Lyudmila Sukhocheva

We study operators of the Keldysh type $A = H(I + S)$ with compact operators H and S. Necessary and sufficient conditions for completeness and basicity of root vectors of A are obtained. A generalization of this type results to Krein spaces will be considered.

Favard's theorem modulo an ideal

Franciszek Szafraniec

This is intended to be a shortcut to the paper

D. Cichoń, J. Stochel and F.H. Szafraniec, Three term recurrence relation modulo an ideal and orthogonality of polynomials of several variables, J. Approx. Theory., to appear.

See http://www.im.uj.edu.pl/badania/preprinty/

A Krein space approach to the Klein-Gordon equation

Christiane Tretter

In this talk an abstract model for the Klein-Gordon equation

$$\left(\left(\frac{\partial}{\partial t} - \mathrm{i}\,eq\right)^2 - \Delta + m^2\right) u = 0$$

in $L^2(\mathbb{R}^n)$ describing the motion of a relativistic particle of mass m and charge e in an electrostatic field with potential q (the velocity of light being normalized to 1) is considered. Three linear operators acting in three different spaces are associated with this model. In order to study their spectral properties, indefinite inner products are introduced. The structure of the spectra of these operators and their relations are investigated.

(joint work with Heinz Langer and Branko Najman†)

Uniformly dissipative perturbations of selfadjoint operators in Krein spaces

Carsten Trunk

Let B be a bounded uniformly dissipative operator and let A be a selfadjoint operator in the Krein space $(\mathcal{H}, [\cdot, \cdot])$ such that $\lambda_0 \in (a, b)$ is no accumulation point of of the non-real spectrum of A and

$$[a, b] \setminus \{\lambda_0\} \subset \sigma_{++}(A) \cup \rho(A).$$

We investigate the spectrum of the operator $A + B$ near λ_0 in a neighbourhood $\mathcal{U}$ of λ_0. If $\lambda_0 \in \sigma_{\pi_+}(A)$ we show that for all uniformly dissipative operators B with $\|B\| < \epsilon$ the spectrum of $A + B$ in $\mathcal{U}$ intersected with the lower half plane consists of finitely many normal eigenvalues. We decribe also the cases $\lambda_0 \in \sigma_{++}(A)$ and $\lambda_0 \notin \sigma_{\pi_+}(A)$.

Here a point λ of the approximative point spectrum $\sigma_{ap}(A)$ of A is called a spectral point of positive type, $\lambda \in \sigma_{++}(A)$, if for every normed approximative eigensequence (x_n) corresponding to λ all accumulation points of the sequence $([x_n, x_n])$ are positive. Spectral points of type π_+ are points of the approximative point spectrum of A which are defined in almost the same way as the points of positive type but we require the property of approximative eigensequences (x_n) mentioned above only for sequences (x_n) belonging to some subspace of finite codimension.

This is a joint work with T. Ya. Azizov and P. Jonas.

Some new results on damped systems (Perturbation bounds and exponential decay)

Kresimir Veselić

We prove a uniform perturbation bound for contractive semi-groups, both in the continuous and in the discrete case. We apply this result to damped second order systems with possibly strongly singular operator coefficients. As a consequence exponential stability for some such systems is proved.

Generalized Schur functions and augmented Schur parameters

Gerald Wanjala

Every Schur function $s(z)$ is the uniform limit of a sequence of finite Blaschke products on compact subsets of the open unit disc. The Blaschke products in the sequence are defined inductively via the Schur parameters of $s(z)$. In the talk we give a similar result for generalized Schur functions.

This is joint work with A. Dijksma.

Extremal extensions of nonnegative operators

Henrik Winkler

The Friedrichs and the Kreĭn-von Neumann extensions of a not necessarily densely defined nonnegative operator S are presented in terms of factorizations. A consequence is the characterization of Ando and Nishio of those nonnegative operators S whose Kreĭn-von Neumann extension is an operator. Furthermore, also the other extremal extensions of S are characterized in terms of factorizations. As an application it is shown that the form sum extension of the sum of two nonnegative selfadjoint operators is an extremal extension.

This is a joint work with S. Hassi, A. Sandovici and H. de Snoo

A variational principle for unbounded block operator matrices and application

Monika Winklmeier

We consider selfadjoint block operator matrices

$$\mathcal{A} = \begin{pmatrix} A & B \\ B^* & D \end{pmatrix}$$

on a Hilbert space $\mathcal{H} = \mathcal{H}_1 \times \mathcal{H}_2$ where D is bounded and A is semibounded. Under further assumptions on B we apply a variational principle to obtain estimates for the eigenvalues of $\mathcal{A}$ to the right of the spectrum of D. We present an application to an example from astro physics and compare our analytic bounds for the eigenvalues to numerical values from the literature.

De Branges spaces of entire functions symmetric about the origin

Harald Woracek

We define and investigate the class of symmetric and the class of semibounded de Branges spaces of entire functions. A construction is made which assigns to each symmetric de Branges space a semibounded one. By employing operator theoretic tools it is shown that every semibounded de Branges space can be obtained in this way, and which symmetric spaces give rise to the same semibounded space. Those subclasses of Hermite-Biehler functions are determined which correspond to symmetric or semibounded, respectively, nondegenerated de Branges spaces. The above assignment is determined in terms of the respective generating Hermite-Biehler functions.

This is a joint work with Michael Kaltenbäck and Henrik Winkler.