Front progression in the East model

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The East model

Continuous time Markov process on $\{0, 1\}^\mathbb{Z}$. 
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Continuous time Markov process on \( \{0, 1\}^\mathbb{Z} \).

Density parameter \( p \in (0, 1) \).
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Generator

$$\mathcal{L} f(\eta) = \sum_{x \in \mathbb{Z}} (1 - \eta_{x+1})(p(1 - \eta_x) + (1 - p)\eta_x)[f(\eta^x) - f(\eta)],$$

where

$$\eta^x_y = \begin{cases} 
1 - \eta_x & \text{if } y = x \\
\eta_y & \text{if } y \neq x 
\end{cases}$$
Graphical construction

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Some properties of the East model

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- Equilibrium measure $\mu = B(p)^{\otimes \mathbb{Z}}$ (reversible)
Some properties of the East model

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\[ \begin{align*}
\text{\begin{array}{c}
\includegraphics[width=0.5\textwidth]{diagram.png}
\end{array}}
\end{align*} \]

- Equilibrium measure \( \mu = \mathcal{B}(p)^{\otimes \mathbb{Z}} \) (reversible)

- Exponential return to equilibrium, but not uniform.
The East model

Out of trouble

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Front progression
The East model Front Out of trouble
Problem

- Start from any configuration with right-most zero at 0.
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- Let the East dynamics run for time $t$.

$X_t$: position of the front (i.e. the right-most zero) at time $t$.

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Questions

- $\frac{X_t}{t} \rightarrow v < 0$?

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What does the front see?
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- What does the front see? Invariant measure for $(\theta(t))_{t \geq 0}$?
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Questions

- $\frac{X_t}{t} \rightarrow v < 0$?
- What does the front see? Invariant measure for $(\theta \eta(t))_{t \geq 0}$? Convergence of $(\theta \eta(t))_{t \geq 0}$?
Trouble?

No attractiveness $\Rightarrow$ No subadditive argument.
Ex: Contact process. $\times \xrightarrow{1} \square$ and $\square \xrightarrow{\lambda \cdot \# \sim \times} \times$.

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Front progression
Theorem (B., 2012)
Results

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- There exists $v < 0$ such that for every initial $\eta$ as above

$$\frac{X_t}{t} \xrightarrow{t \to \infty} v \quad \text{in probability}.$$
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- There exists $v < 0$ such that for every initial $\eta$ as above
  \[ \frac{X_t}{t} \xrightarrow{t \to \infty} v \quad \text{in probability}. \]

- The process seen from the front has a unique invariant measure $\nu$ and
  \[ \theta \eta(t) \xrightarrow{} \nu \quad \text{in distribution}. \]
Central argument

Far from the front, $\theta \eta(t)$ is almost distributed as $\mu$. 
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\[
X_t \sim \nu_{t;L,M}^{\eta} ; L,M - \mu
\]
Central argument

Far from the front, \( \theta \eta(t) \) is almost distributed as \( \mu \).

Theorem (B., 2012)

- If \( L + M \leq Ct \)
  \[
  \| \nu_{t;L,M}^{\eta} - \mu \|_{TV} \leq e^{-\epsilon L}
  \]
**Central argument**

Far from the front, $\theta \eta(t)$ is almost distributed as $\mu$.

**Theorem (B., 2012)**

- If $L + M \leq Ct$
  \[ \| \nu^{\eta}_{t;L,M} - \mu \|_{TV} \leq e^{-\epsilon L} \]

- If $L + M > Ct$ and $\eta$ has "enough zeros"
  \[ \| \nu^{\eta}_{t;L,M} - \mu \|_{TV} \leq e^{-\epsilon (L \wedge t)} \]
Thank you!