Random Media (RM 2013)

TU Berlin, September 16 – 17, 2013

Workshop program
### Monday, 16. September: **Room MA043**

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| 09:30 | Takashi Kumagai
Biased random walk on critical Galton-Watson trees conditioned to survive |
| 10:30 | **Coffee Break**                                                     |
| 11:00 | Xiaoquin Guo
Einstein relation for random walks on percolation clusters        |
| 12:00 | Artem Sapozhnikov
Distances in percolation models with long-range correlations          |
| 13:00 | **Lunch (Self Catered)**                                            |
| 14:30 | Marek Biskup
Extreme local extrema of two-dimensional discrete Gaussian Free Field |
| 15:30 | **Coffee Break**                                                     |
| 16:00 | Itai Benjamini
Random walk on planar graphs                                       |
| 17:00 | Noam Berger
Local limit theorem for ballistic random walk in random environments |
| 18:30 | **Reception**                                                        |

### Tuesday, 17. September: **Room MA043**

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| 09:00 | Martin Barlow
Quenched and averaged invariance principles for the random conductance model |
| 10:00 | Luca Avena
Recent progresses in random walks in dynamical random environments |
| 10:30 | **Coffee Break**                                                     |
| 11:00 | Daniel Marahrens
Annealed estimates on the Green’s function                          |
| 12:00 | Pierre Mathieu
The Einstein relation for reversible diffusions in a random environment |
| 13:00 | **Lunch (Self Catered)**                                            |
| 14:30 | Moustapha Ba
Sobolev inequality and individual invariance principle for diffusions in periodic potentials |
| 15:00 | Hirofumi Osada
Dynamical rigidity of stochastic Coulomb systems in infinite-dimensions |
| 16:00 | **Conclusion of the Workshop**                                       |
Speaker Abstracts

Luca Avena (WIAS Berlin)
Recent progresses in random walks in dynamical random environments

We consider random walks in Markovian evolving environments on the integer lattice. We review recent results based on different projects with several coauthors. Particular emphasis will be given to fast uniform mixing and slowly non-uniform mixing types of environments.

Moustapha Ba (Université d’Aix-Marseille)
Sobolev inequality and individual invariance principle for diffusions in periodic potentials

We prove here, using stochastic analysis methods; the invariance principle for a $\mathbb{R}^d$-diffusions $d \geq 2$; involving in periodic potential beyond uniform boundedness assumptions. The potential is not assumed to have any regularity. So the stochastic calculus theory for processes associated to Dirichlet forms used to justify the existence and uniqueness of this process starting for almost all $x \in \mathbb{R}^d$; allows us to show an invariance principle for almost all starting point (individual invariance principle). For that, we show then one Sobolev inequality to bound the probability of transition associated to the time changed diffusion for times large enough and deduce the existence of one bounded density. This property allows us to prove the tightness of the sequence of processes in the uniform topology; the proof of the convergence in finite dimensional distribution is very standard: construction of corrector and central limit theorem for martingale with continuous time. This is joint work with Pierre Mathieu.

Martin Barlow (University of British Columbia)
Quenched and averaged invariance principles for the random conductance model

The random conductance model considers a discrete or continuous time random walk on $\mathbb{Z}^d$ where the edges $e$ are given random weights $\mu_e$. While the case of i.i.d. $\mu_e$ is now essentially solved, many questions remain for stationary ergodic environments. I will discuss the difference between ‘quenched’ and ‘annealed’ (or, better, ‘averaged’) invariance principles in this context. This is joint work with K. Burdzy, A. Timar.

Itai Benjamini (Weizmann Institute of Science)
Random walk on planar graphs

We will discuss some examples of planar graphs and the behavior of random walk on them.

Noam Berger (TU München and the Hebrew University of Jerusalem)
Local limit theorem for ballistic random walk in random environments

We prove a version of a local CLT for random walk in random environments satisfying Sznitman’s ballisticity condition (T’) in dimension four and higher. This is joint work with Moran Cohen and Ron Rosenthal.
Marek Biskup (UCLA)

Extreme local extrema of two-dimensional discrete Gaussian Free Field

For the two-dimensional discrete Gaussian Free Field in a box of side $N$, we describe the statistics of the values and spatial locations of the nearly-maximal locally-extreme points in the limit as $N$ tends to infinity. The result turns out to be a Gumbel-type space-time point process with a random (spatial) intensity measure which exhibits a host of interesting properties. Connections with the subject of random walk in random environment will be made. Based on ongoing project with Oren Louidor.

Xiaojin Guo (TU München)

Einstein relation for random walks on percolation clusters

The Einstein relation describes the relation between the response of a system to a perturbation and its diffusivity at equilibrium. It states that the derivative of the velocity (with respect to the strength of the perturbation) equals the diffusivity. In this talk we will discuss the Einstein relation for biased random walks on supercritical percolation clusters in $\mathbb{Z}^d$, $d \geq 2$. This talk is based on an ongoing project with Noam Berger, Nina Gantert and Jan Nagel.

Takashi Kumagai (Kyoto University)

Biased random walk on critical Galton-Watson trees conditioned to survive

We consider the biased random walk on a critical Galton-Watson tree conditioned to survive, and confirm that this model with trapping belongs to the same universality class as certain one-dimensional trapping models with slowly-varying tails. Indeed, in each of these two settings, it is possible to establish closely-related functional limit theorems involving an extremal process and also demonstrate extremal aging occurs. This is a joint work with David Croydon (Warwick) and Alexander Fribergh (Toulouse).

Daniel Marahrens (MPI Leipzig)

Annealed estimates on the Green’s function

We consider the elliptic Green’s function $G$ of a random walk in a random environment. We present a method to upgrade optimal annealed bounds on the first moments (i.e. the $L^1$-norm in probability) of $G$ to all (finite) moments via a logarithmic Sobolev inequality. We also present some consequences of these estimates, including estimates for the (random part of the) homogenization error. This is joint work with Felix Otto.

Pierre Mathieu (Université d’Aix-Marseille)

The Einstein relation for reversible diffusions in a random environment

We showed the Einstein relation for perturbations of reversible diffusions in a random environment with finite range correlations. I will discuss the main ideas of the proof and the motivation underlying the Einstein relation. The talk is based on joint work with Nina Gantert and Andrey Piatnitski.
Hirofumi Osada (Kyushu University)

Dynamical rigidity of stochastic Coulomb systems in infinite-dimensions

Stochastic Coulomb dynamics in infinite-dimensions are infinitely many Brownian particles in \( \mathbb{R}^d \) interacting via \( \gamma \)-dimensional Coulomb potential \( \Psi_\gamma \) with inverse temperature \( \beta \). When the systems are translation invariant, then the dynamics are given by the infinite-dimensional stochastic differential equations

\[
dX^i_t = dB^i_t - \frac{\beta}{2} \lim_{r \to \infty} \sum_{j \neq i, |X^i_t - X^j_t| < r} \nabla \Psi_\gamma(X^i_t - X^j_t) \, dt
\]

We suppose \( d \leq \gamma \leq d + 2 \) because \( \Psi_\gamma \) become Ruelle class potentials if \( \gamma > d + 2 \), and it seems difficult to justify the corresponding SDEs for \( \gamma < d \). If \( d \leq \gamma \leq d + 2 \), then we call the above SDEs (translation invariant) stochastic Coulomb dynamics whereas for \( \gamma = d \) it is called strict stochastic Coulomb dynamics. So far the only example of strict stochastic Coulomb dynamics is the Ginibre interacting Brownian motions, namely the case \( (\beta, \gamma, d) = (2, 2, 2) \). Dyson’s models and Airy Interacting Brownian motions are examples of stochastic Coulomb dynamics in one dimension with two-dimensional Coulomb potentials. Namely, \( (\beta, \gamma, d) = (1, \gamma, d), (2, \gamma, d), (4, \gamma, d) \).

In this talk, I present various dynamical rigidity of the Ginibre interacting Brownian motions. In particular, I prove that the tagged particles of Ginibre interacting Brownian motions are \textit{sub-diffusive}.

Such a sub-diffusivity of tagged particles may be surprising. For the tagged particles of interacting Brownian motions in \( \mathbb{R}^d \) with \( d \geq 2 \) with Ruelle’s class potentials are always diffusive. This diffusivity is proved for Ruelle’s class potentials with convex hard cores. In case of no hard core, this has been not yet fully proved, but generally believed by the specialists in mathematics, and papers in physics are treated as a fact.

In addition, I give a phase transition conjecture on the rigidity of strict stochastic Coulomb dynamics with the inverse temperature \( \beta \) for general dimensions \( d \geq 2 \). The proof is based on the results of stochastic geometry.

Artem Sapozhnikov (MPI Leipzig)

Distances in percolation models with long-range correlations

Let \( S \) be a random subgraph of \( \mathbb{Z}^d \). I will discuss a set of conditions on the distribution of \( S \) under which the long-scale behavior of distances in \( S \) is comparable to the one in \( \mathbb{Z}^d \). Examples of models satisfying the conditions include Bernoulli percolation, random interlacements and its vacant set, level sets of the Gaussian free field. Based on a joint work with A. Drewitz and B. Ráth.