

Alle Funktionen (Blatt 8 Tutorium 1 und 2)

```
In[1]:= f[x_, y_] = {x^2 + 2 y^2, x^2 - 2 y^2, x^2, -x^2 - 2 y^2,
x^2 + y^4, x^2 - y^4, 3 x^2 y - y^3, (4 x^2 + y^2) E^(-x^2 - 4 y^2) }

Out[1]= {x^2 + 2 y^2, x^2 - 2 y^2, x^2, -x^2 - 2 y^2, x^2 + y^4, x^2 - y^4, 3 x^2 y - y^3, e^-x^2 - 4 y^2 (4 x^2 + y^2)}
```

■ Gradienten

```
In[2]:= {D[#, x], D[#, y]} & /@ f[x, y] // Simplify

Out[2]= {{2 x, 4 y}, {2 x, -4 y}, {2 x, 0}, {-2 x, -4 y}, {2 x, 4 y^3}, {2 x, -4 y^3},
{6 x y, 3 (x^2 - y^2)}, {-2 e^-x^2 - 4 y^2 x (-4 + 4 x^2 + y^2), -2 e^-x^2 - 4 y^2 y (-1 + 16 x^2 + 4 y^2)}}
```

■ Hessesche Matrizen (Aufgabe 1 d.h. die 8. Matrix ist unten nochmal)

```
In[3]:= MatrixForm[Simplify[{D[#, {x, 2}],
D[D[#, x], y]}, {D[D[#, x], y], D[#, {y, 2}]}}] & /@
f[x, y][[{1, 2, 3, 4, 5, 6, 7, 8}]]]

Out[3]= {{(2, 0), (2, 0), (2, 0), (-2, 0), (2, 0),
(2, 0), (6 y, 6 x), (e^-x^2 - 4 y^2 (16
4 e^-x^2
```

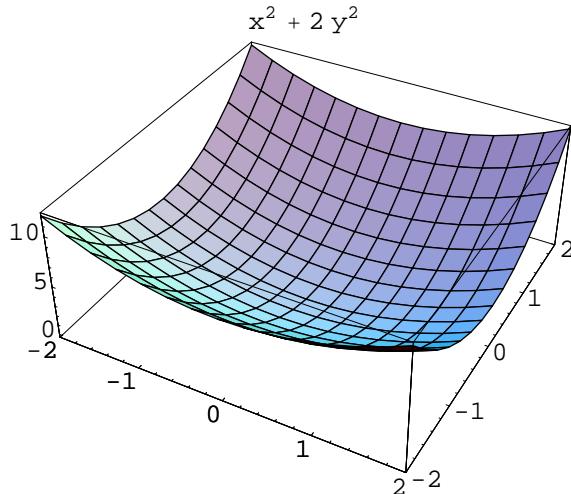
Aufgabe 2

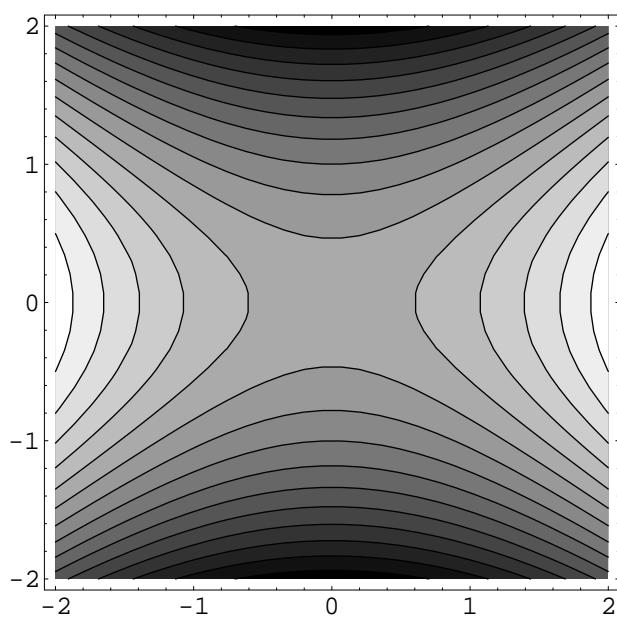
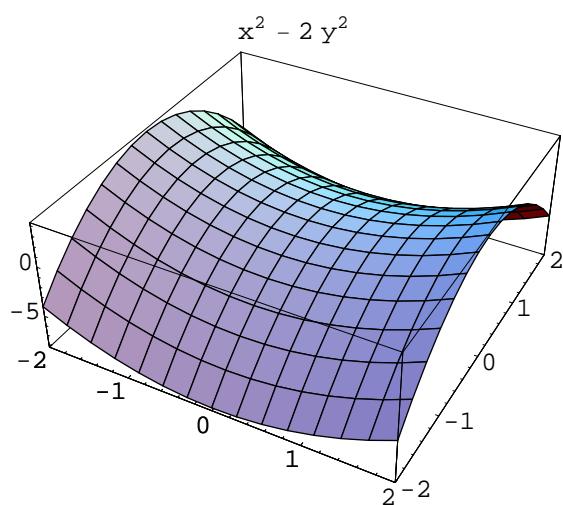
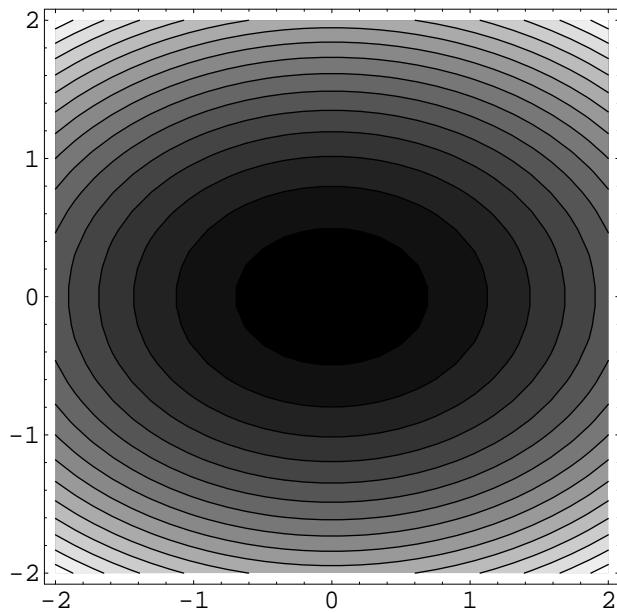
```
In[4]:= ?Plot3D ContourPlot

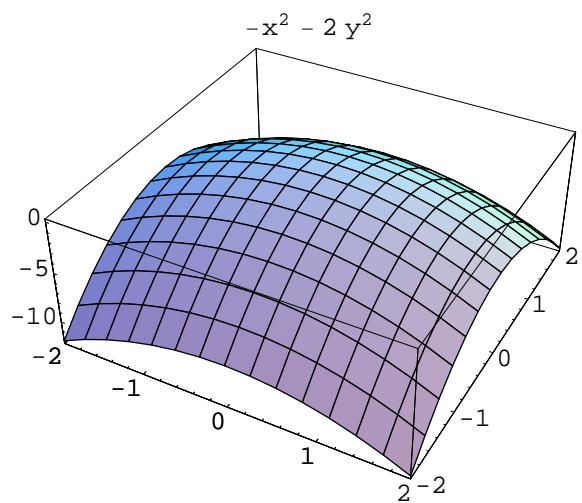
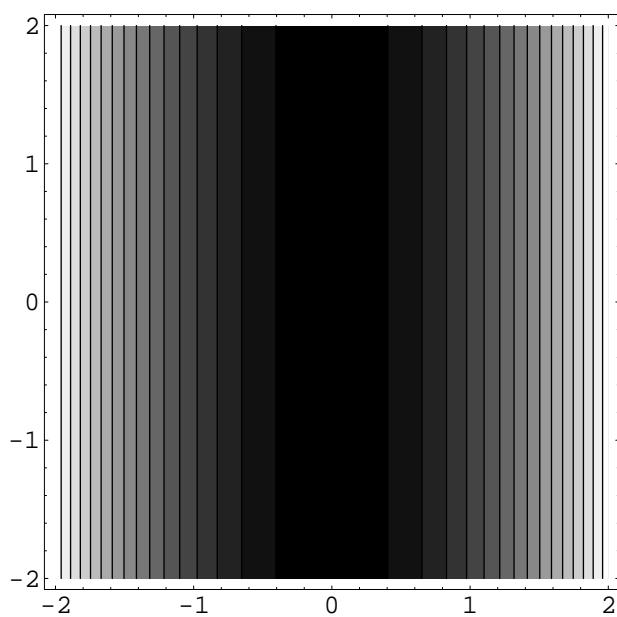
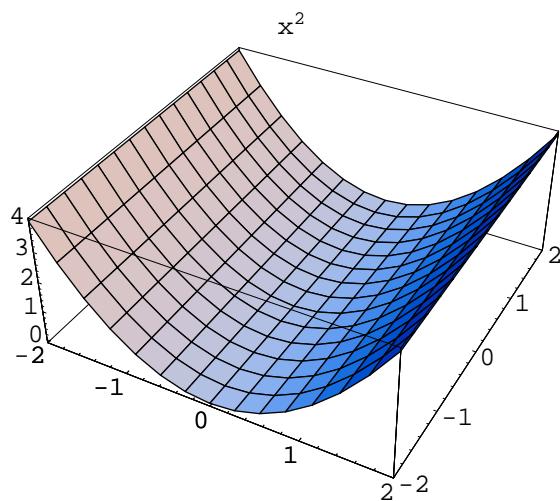
Plot3D[f, {x, xmin, xmax}, {y, ymin, ymax}] generates a three-
dimensional plot of f as a function of x and y. Plot3D[{f, s}, {x, xmin,
xmax}, {y, ymin, ymax}] generates a three-dimensional plot in which the
height of the surface is specified by f, and the shading is specified by s.

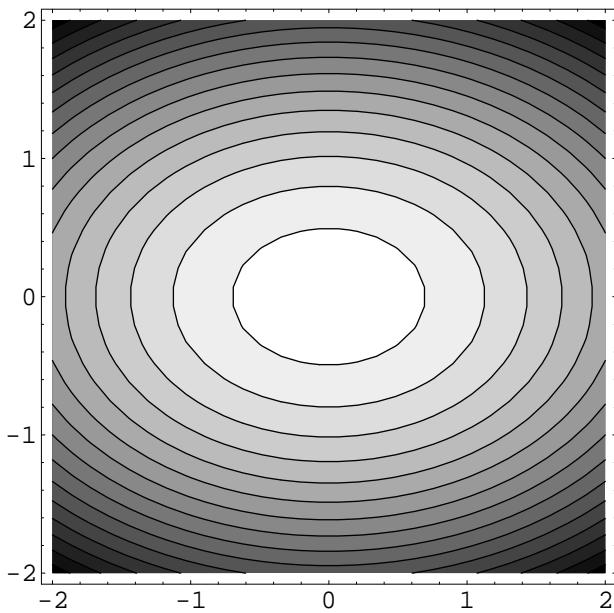
ContourPlot[f, {x, xmin, xmax}, {y, ymin,
ymax}] generates a contour plot of f as a function of x and y.
```

```
In[5]:= {Plot3D[#, {x, -2, 2}, {y, -2, 2}, PlotRange -> All, PlotLabel -> #],
ContourPlot[#, {x, -2, 2}, {y, -2, 2}, Contours -> 15,
PlotPoints -> 30] & /@ (f[x, y][[{1, 2, 3, 4}]]);
```

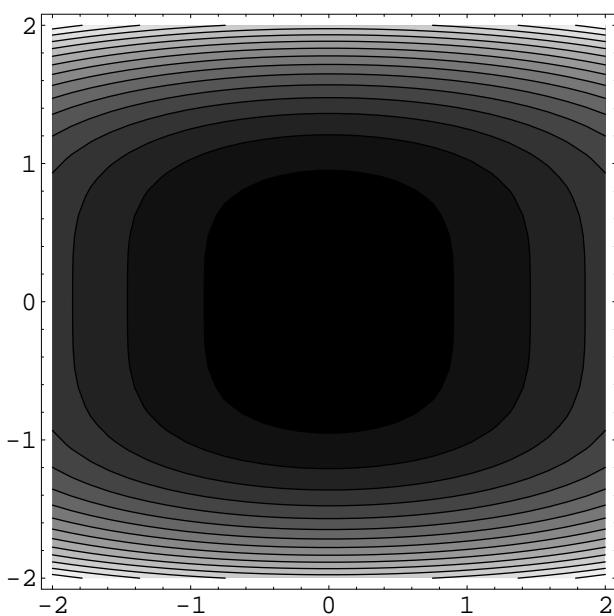
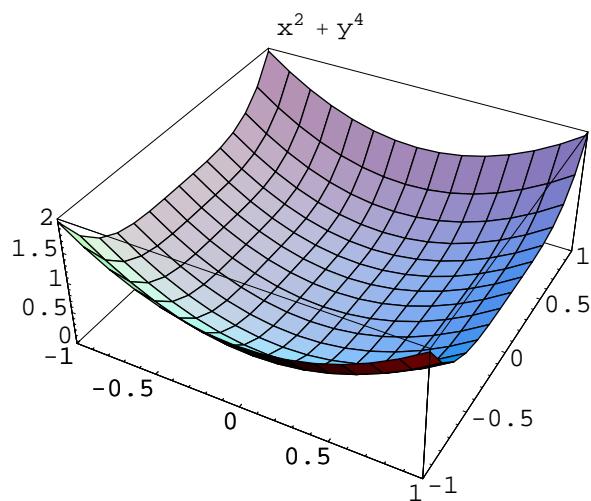


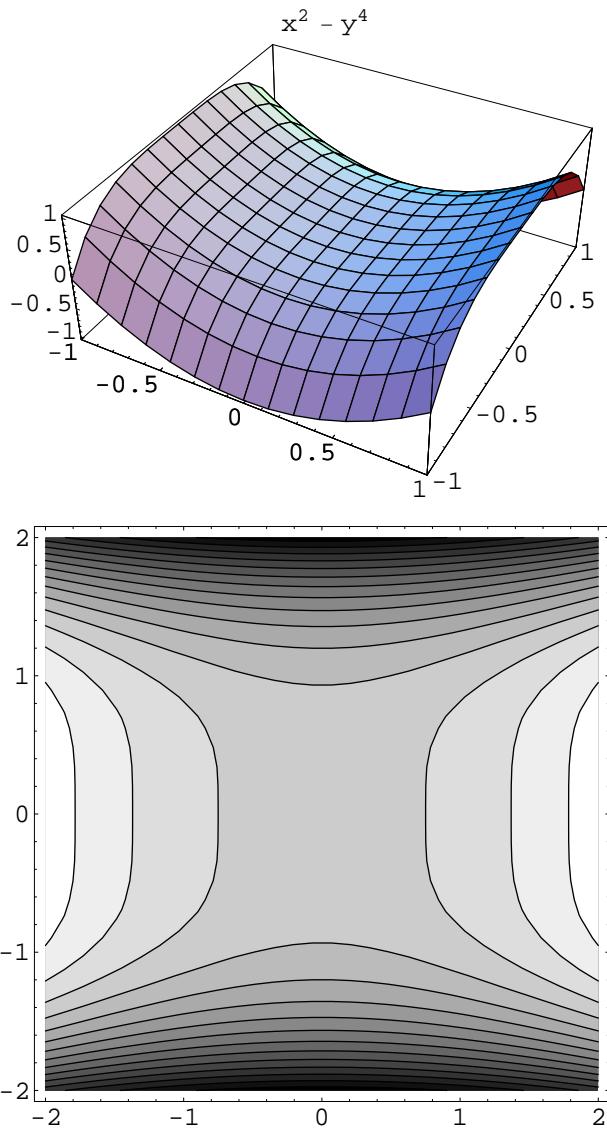






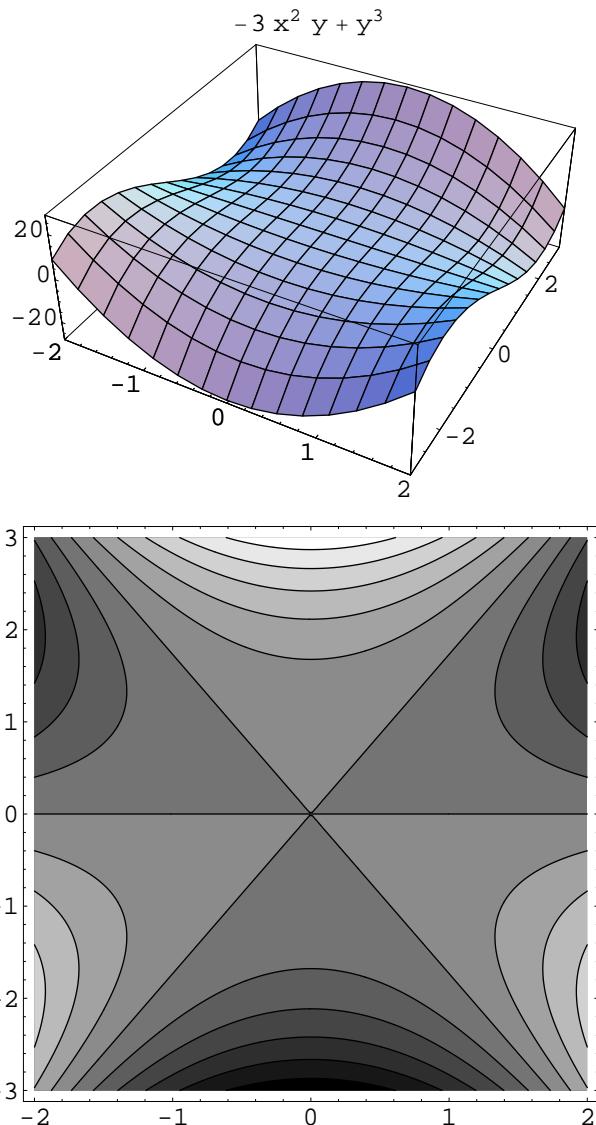
```
In[6]:= {Plot3D[#, {x, -1, 1}, {y, -1, 1}, PlotRange -> All, PlotLabel -> #] , ContourPlot[#, {x, -2, 2}, {y, -2, 2}, Contours -> 15, PlotPoints -> 30]} & /@ (f[x, y] [[{5, 6}]]);
```





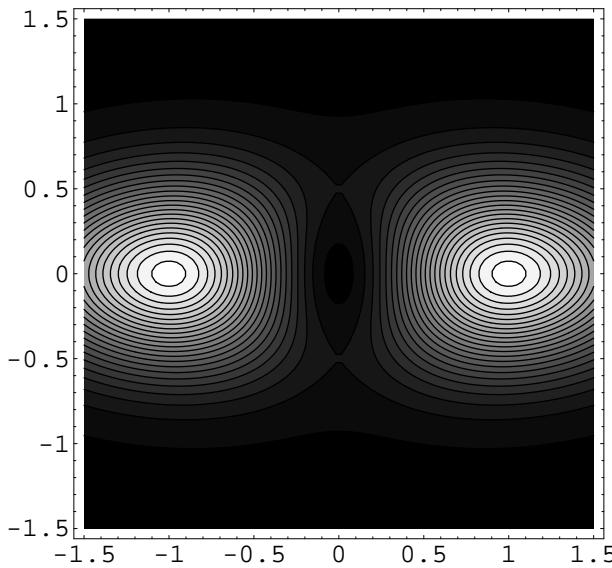
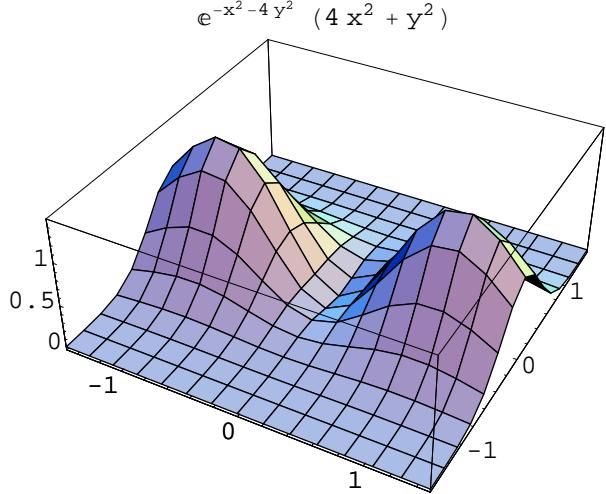
■ "Affensattel"

```
In[7]:= Plot3D[f[x, -y][[7]], {x, -2, 2}, {y, -3, 3},
  PlotRange -> All, PlotLabel -> f[x, -y][[7]]];
ContourPlot[f[x, -y][[7]], {x, -2, 2}, {y, -3, 3}, PlotPoints -> 200, Contours -> 11];
```



Aufgabe 1

```
In[8]:= Plot3D[f[x, y][[8]], {x, -1.5, 1.5}, {y, -1.5, 1.5},
  PlotRange -> All, PlotLabel -> f[x, y][[8]]]; ContourPlot[f[x, y][[8]],
  {x, -1.5, 1.5}, {y, -1.5, 1.5}, PlotPoints -> 100, Contours -> 23];
```



```
In[9]:= g[x_, y_] = f[x, y][[8]] (* x^4+y^4-(x+y)^2 *)
```

```
Out[9]= e^{-x^2-4y^2} (4x^2 + y^2)
```

```
In[10]:= gradg[x_, y_] = {D[g[x, y], x], D[g[x, y], y]}
```

```
Out[10]= {8 e^{-x^2-4y^2} x - 2 e^{-x^2-4y^2} x (4x^2 + y^2), 2 e^{-x^2-4y^2} y - 8 e^{-x^2-4y^2} y (4x^2 + y^2)}
```

```
In[11]:= hess[x_, y_] = MatrixForm[Simplify[{{D[g[x, y], {x, 2}], D[D[g[x, y], x], y]}, {D[D[g[x, y], x], y], D[g[x, y], {y, 2}]}]]]
```

```
Out[11]//MatrixForm=
```

$$\begin{pmatrix} e^{-x^2-4y^2} (16x^4 + 4x^2(-10 + y^2) - 2(-4 + y^2)) & 4e^{-x^2-4y^2} xy (-17 + 16x^2 + 4y^2) \\ 4e^{-x^2-4y^2} xy (-17 + 16x^2 + 4y^2) & e^{-x^2-4y^2} (2 - 40y^2 + 64y^4 + 32x^2(-1 + 8y^2)) \end{pmatrix}$$

```
In[12]:= GleichungKritischePunkte = Simplify[gradg[x0, y0] E^(x0^2 + 4y0^2) == 0]
```

```
Out[12]= {-2x0 (-4 + 4x0^2 + y0^2), -2y0 (-1 + 16x0^2 + 4y0^2)} == 0
```

In[13]:= **kritischePunkte** = Solve[GleichungKritischePunkte]

$$\text{Out}[13]= \left\{ \{x0 \rightarrow -1, y0 \rightarrow 0\}, \{x0 \rightarrow 0, y0 \rightarrow 0\}, \{x0 \rightarrow 1, y0 \rightarrow 0\}, \{y0 \rightarrow -\frac{1}{2}, x0 \rightarrow 0\}, \{y0 \rightarrow \frac{1}{2}, x0 \rightarrow 0\} \right\}$$

In[14]:= **?Series**

Series[f, {x, x0, n}] generates a power series expansion for f about the point x = x0 to order (x - x0)^n. Series[f, {x, x0, nx}, {y, y0, ny}] successively finds series expansions with respect to y, then x.

In[15]:= **Series[g[x, y], {x, x0, 2}, {y, y0, 2}] /. kritischePunkte**

$$\begin{aligned} \text{Out}[15]= & \left\{ \left(\frac{4}{e} - \frac{15y^2}{e} + O[y]^3 \right) + \left(\frac{2y^2}{e} + O[y]^3 \right) (x+1) + \left(-\frac{8}{e} + \frac{33y^2}{e} + O[y]^3 \right) (x+1)^2 + O[x+1]^3, \right. \\ & (y^2 + O[y]^3) + O[y]^3 x + (4 - 17y^2 + O[y]^3) x^2 + O[x]^3, \\ & \left(\frac{4}{e} - \frac{15y^2}{e} + O[y]^3 \right) + \left(-\frac{2y^2}{e} + O[y]^3 \right) (x-1) + \left(-\frac{8}{e} + \frac{33y^2}{e} + O[y]^3 \right) (x-1)^2 + O[x-1]^3, \\ & \left. \left(\frac{1}{4e} - \frac{2(y+\frac{1}{2})^2}{e} + O[y+\frac{1}{2}]^3 \right) + O[y+\frac{1}{2}]^3 x + \right. \\ & \left(\frac{15}{4e} + \frac{16(y+\frac{1}{2})}{e} + \frac{18(y+\frac{1}{2})^2}{e} + O[y+\frac{1}{2}]^3 \right) x^2 + O[x]^3, \\ & \left. \left(\frac{1}{4e} - \frac{2(y-\frac{1}{2})^2}{e} + O[y-\frac{1}{2}]^3 \right) + O[y-\frac{1}{2}]^3 x + \right. \\ & \left. \left(\frac{15}{4e} - \frac{16(y-\frac{1}{2})}{e} + \frac{18(y-\frac{1}{2})^2}{e} + O[y-\frac{1}{2}]^3 \right) x^2 + O[x]^3 \right\} \end{aligned}$$

In[16]:= **gradg[x0, y0] /. kritischePunkte**

$$\text{Out}[16]= \{\{0, 0\}, \{0, 0\}, \{0, 0\}, \{0, 0\}, \{0, 0\}\}$$

In[17]:= **hess[x0, y0] /. kritischePunkte**

$$\text{Out}[17]= \left\{ \begin{pmatrix} -\frac{16}{e} & 0 \\ 0 & -\frac{30}{e} \end{pmatrix}, \begin{pmatrix} 8 & 0 \\ 0 & 2 \end{pmatrix}, \begin{pmatrix} -\frac{16}{e} & 0 \\ 0 & -\frac{30}{e} \end{pmatrix}, \begin{pmatrix} \frac{15}{2e} & 0 \\ 0 & -\frac{4}{e} \end{pmatrix}, \begin{pmatrix} \frac{15}{2e} & 0 \\ 0 & -\frac{4}{e} \end{pmatrix} \right\}$$