

Loesungen zum 8. Blatt

1. Aufgabe

In[1]:= f[x_, y_, z_] := y/x + z/y - x/z

In[2]:= g[x_, y_, z_] := x Sqrt[z]/y

■ Gradienten

In[3]:= {D[f[x, y, z], x], D[f[x, y, z], y], D[f[x, y, z], z]} // Simplify // MatrixForm
{D[g[x, y, z], x], D[g[x, y, z], y], D[g[x, y, z], z]} // Simplify // MatrixForm

Out[3]//MatrixForm=

$$\begin{pmatrix} -\frac{y}{x^2} & -\frac{1}{z} \\ \frac{1}{x} & -\frac{z}{y^2} \\ \frac{1}{y} & +\frac{x}{z^2} \end{pmatrix}$$

Out[4]//MatrixForm=

$$\begin{pmatrix} \frac{\sqrt{z}}{y} \\ -\frac{x\sqrt{z}}{y^2} \\ \frac{x}{2y\sqrt{z}} \end{pmatrix}$$

■ Hessesche Matrizen

In[5]:= {{D[f[x, y, z], {x, 2}], D[D[f[x, y, z], x], y], D[D[f[x, y, z], x], z]}, {D[D[f[x, y, z], y], x], D[f[x, y, z], {y, 2}], D[D[f[x, y, z], y], z]}, {D[D[f[x, y, z], z], x], D[D[f[x, y, z], z], y], D[f[x, y, z], {z, 2}]}} // Simplify // MatrixForm
{{D[g[x, y, z], {x, 2}], D[D[g[x, y, z], x], y], D[D[g[x, y, z], x], z]}, {D[D[g[x, y, z], y], x], D[g[x, y, z], {y, 2}], D[D[g[x, y, z], y], z]}, {D[D[g[x, y, z], z], x], D[D[g[x, y, z], z], y], D[g[x, y, z], {z, 2}]}} // Simplify // MatrixForm

Out[5]//MatrixForm=

$$\begin{pmatrix} \frac{2y}{x^3} & -\frac{1}{x^2} & \frac{1}{z^2} \\ -\frac{1}{x^2} & \frac{2z}{y^3} & -\frac{1}{y^2} \\ \frac{1}{z^2} & -\frac{1}{y^2} & -\frac{2x}{z^3} \end{pmatrix}$$

Out[6]//MatrixForm=

$$\begin{pmatrix} 0 & -\frac{\sqrt{z}}{y^2} & \frac{1}{2y\sqrt{z}} \\ -\frac{\sqrt{z}}{y^2} & \frac{2x\sqrt{z}}{y^3} & -\frac{x}{2y^2\sqrt{z}} \\ \frac{1}{2y\sqrt{z}} & -\frac{x}{2y^2\sqrt{z}} & -\frac{x}{4yz^{3/2}} \end{pmatrix}$$

2.Aufgabe

In[7]:= $f[x_, y_] = x^4 + y^4 - (x + y)^2$

Out[7]= $x^4 + y^4 - (x + y)^2$

■ Gradient

In[8]:= (gradf[x_, y_] = {D[f[x, y], x], D[f[x, y], y]} // Simplify) // MatrixForm

Out[8]//MatrixForm=

$$\begin{pmatrix} 4x^3 - 2(x + y) \\ 4y^3 - 2(x + y) \end{pmatrix}$$

■ kritische Punkte

In[9]:= kritischePunkte = Solve[gradf[x0, y0] == 0] // Simplify

Out[9]= $\left\{ \{x0 \rightarrow -1, y0 \rightarrow -1\}, \{x0 \rightarrow 0, y0 \rightarrow 0\}, \{x0 \rightarrow 0, y0 \rightarrow 0\}, \{x0 \rightarrow 0, y0 \rightarrow 0\}, \{x0 \rightarrow 1, y0 \rightarrow 1\}, \{x0 \rightarrow -\frac{1}{4} i \sqrt{1-i \sqrt{3}} (-i+\sqrt{3}), y0 \rightarrow \frac{1}{2} \sqrt{1-i \sqrt{3}}\}, \{x0 \rightarrow \frac{1}{4} \sqrt{1-i \sqrt{3}} (1+i \sqrt{3}), y0 \rightarrow -\frac{1}{2} \sqrt{1-i \sqrt{3}}\}, \{x0 \rightarrow \frac{1}{4} (1-i \sqrt{3}) \sqrt{1+i \sqrt{3}}, y0 \rightarrow -\frac{1}{2} \sqrt{1+i \sqrt{3}}\}, \{x0 \rightarrow \frac{1}{4} i \sqrt{1+i \sqrt{3}} (i+\sqrt{3}), y0 \rightarrow \frac{1}{2} \sqrt{1+i \sqrt{3}}\} \right\}$

■ Taylorpolynome

In[10]:= TP = Series[f[x, y], {x, x0, 2}, {y, y0, 2}] /. kritischePunkte[[{1, 2, 5}]] // Normal

Out[10]= $\{-2 + 5(1+x)^2 - 2(1+x)(1+y) + 5(1+y)^2, -x^2 - 2xy - y^2, -2 + 5(-1+x)^2 - 2(-1+x)(-1+y) + 5(-1+y)^2\}$

In[11]:= ContourPlot[#, {x, -3, 3}, {y, -3, 3}, PlotLabel → #, Contours → 15, PlotPoints → 30] & /@ TP;

