The MIP-Solving-Framework SCIP

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DFG Research Center MATHEON
Mathematics for key technologies

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Definition MIP

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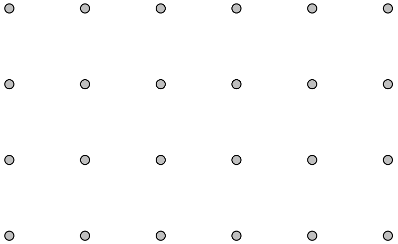
(mixed integer program).



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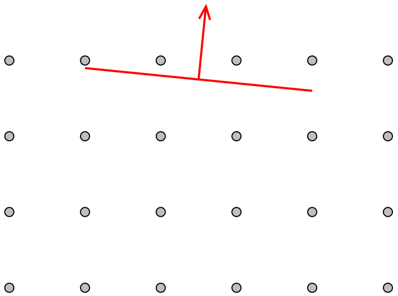

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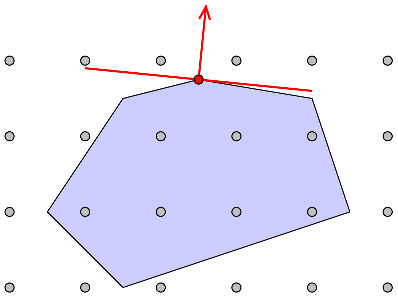

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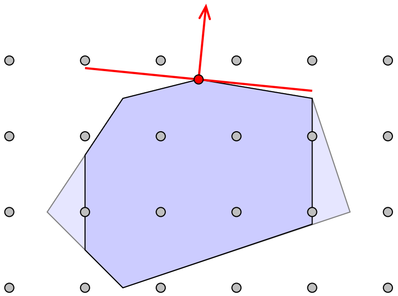

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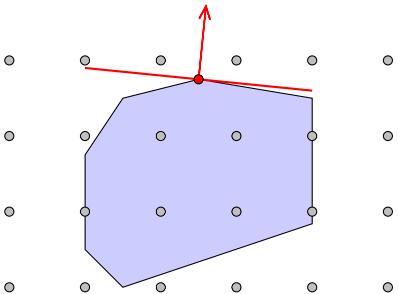

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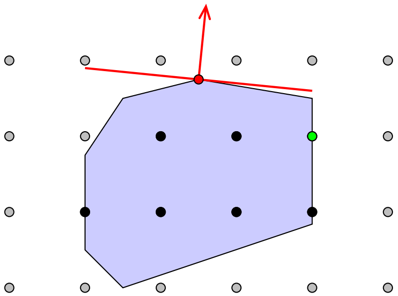
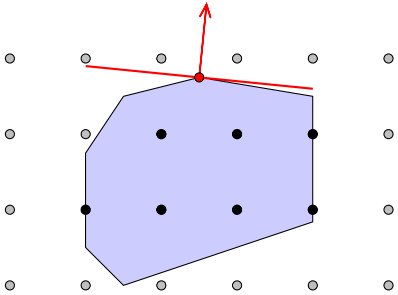


Exact methods

- Branch-And-Bound
- Cutting planes
- Combination: Branch-And-Cut



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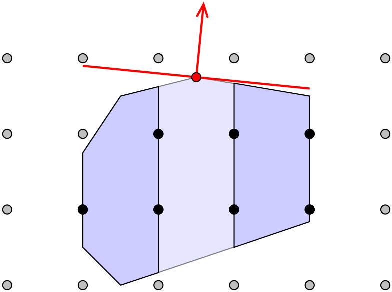

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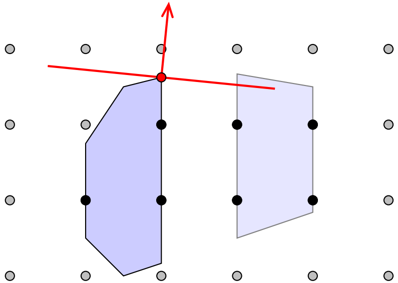

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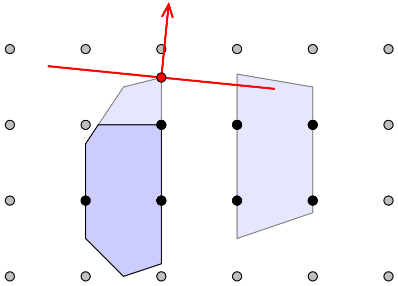

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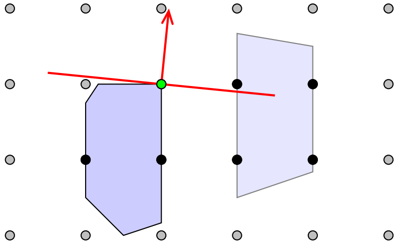

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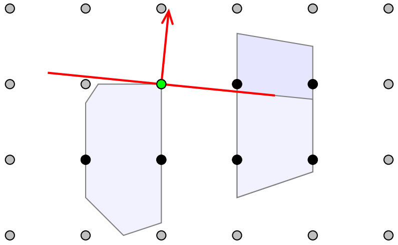

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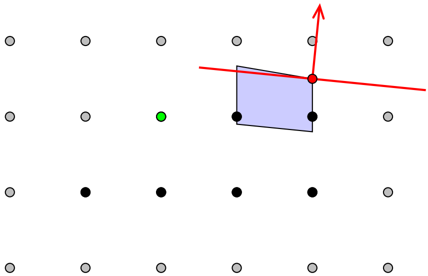

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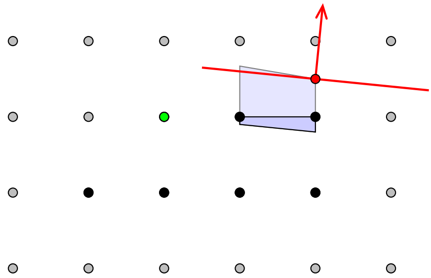

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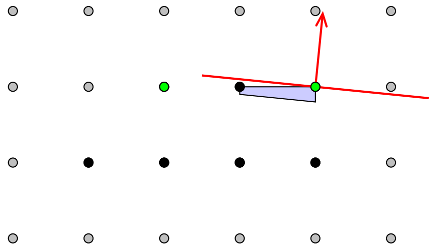

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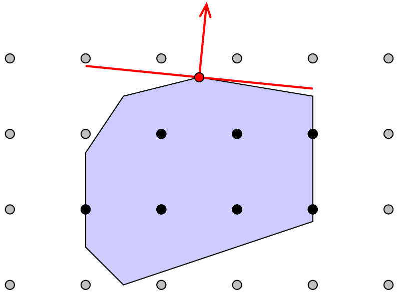

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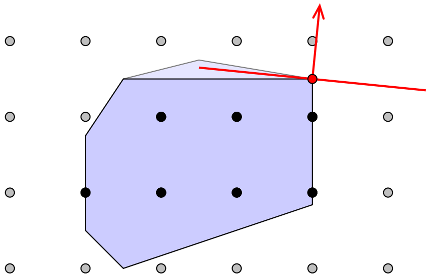

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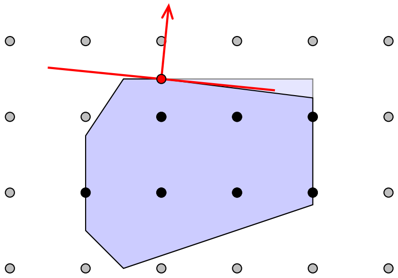

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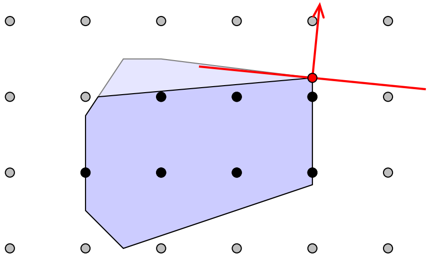

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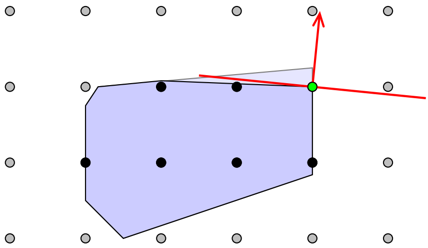

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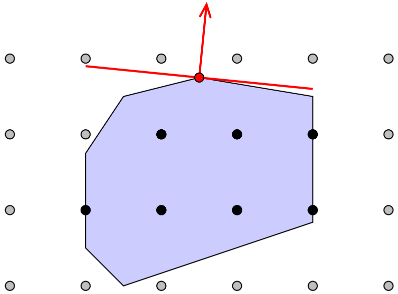

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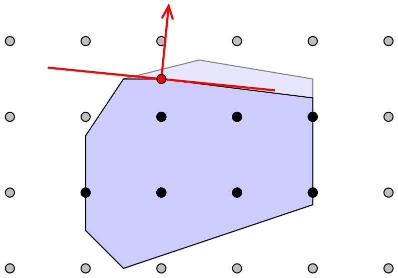

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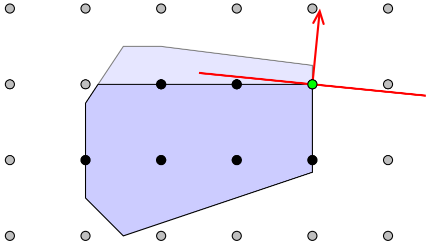

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- General constraints
- Integer variables $\Rightarrow$ (CIP)

Examples

How do we solve CPs?

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- TSP as CP
- n-Queens

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How do we solve CPs?

- Branching:
  - Divide into subproblems, solve recursively
- Domain Propagation:
  - Reductions in variables' domains "propagate"
  - E.g. $x_1 + 2x_2 \geq 5,\ x_1 \leq 2 \ \Rightarrow\ x_2 \geq 2$

SCIP ← CP+MIP

- SCIP combines technologies
- Standalone-solver for MIP
  - A bundle of MIP-solving-components as default plugins
  - MIP-solver as fast as CPlex 9.0
  - Underlying LP-solver: treated as blackbox
- Branch-Cut-And-Price-Framework for MIP and CIP
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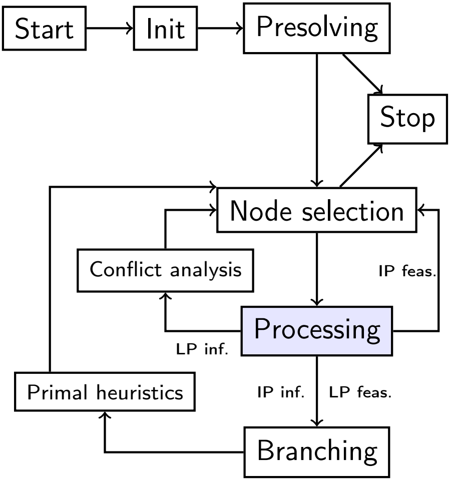
- Pricer: allows dynamic generation of variables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Presolving
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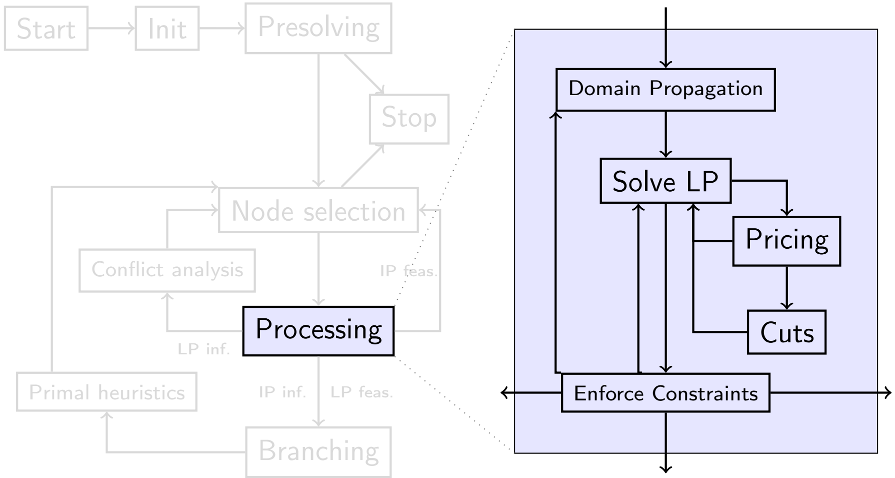
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Domain Propagation
Solve LP
Pricing
Cuts
Enforce Constraints

Default plugins

- 5 presolvers
- 5 node selection rules
- 14 constraint handlers
- 8 separators
- 8 branching rules
- 4 conflict handlers
- 2 propagators
- 23 primal heuristics

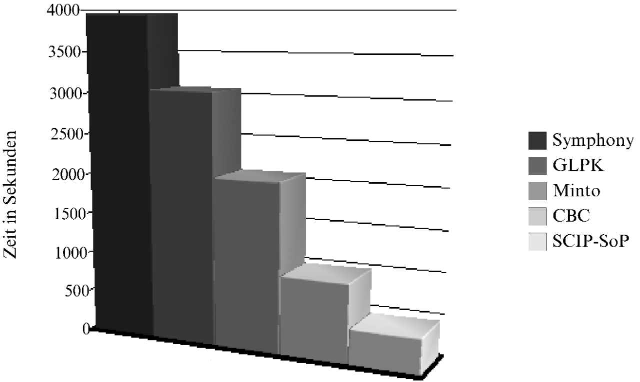
SCIP as a framework for a TSP-solver

Default plugins

SCIP as a framework for a TSP-solver

main program:	196 lines
TSP file reader:	407 lines
graph structure:	80 lines
subtour constraint:	793 lines
Gomory-Hu algo.:	658 lines
FarInsert heuristic:	354 lines
2-Opt heuristic:	304 lines
altogether:	2792 lines

Comparison With Other Free MIP-Solvers



Characteristics

- Highest priority to feasibility
- Distinguish:
  - Start heuristics
  - Improvement heuristics
- Keep control of effort!
- Use available information

Used Information

- **Variables' locking numbers:**
  Potentially violated rows
- **Variables' pseudocosts:**
  Average objective change
- **Special points:**
  - LP optimum at root node
  - Current LP optimum
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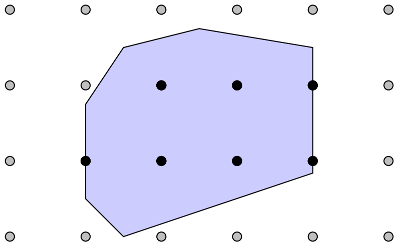

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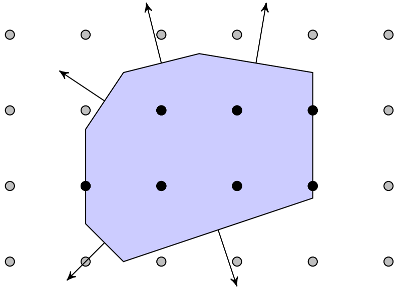

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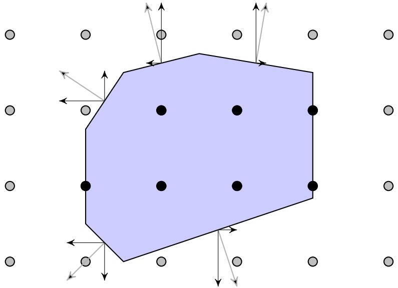

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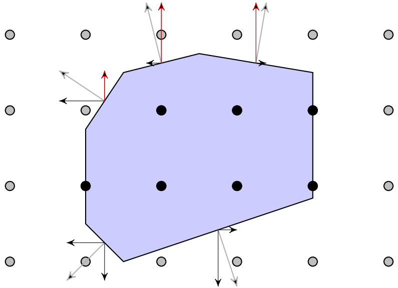

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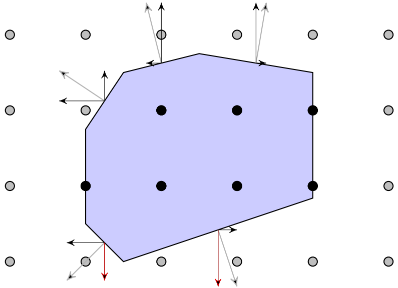

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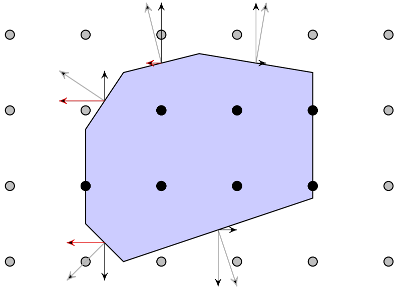

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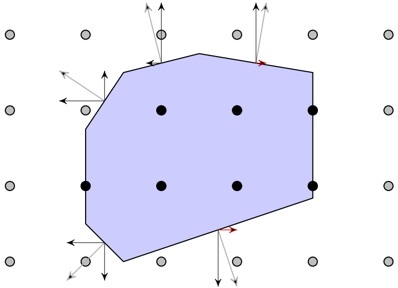

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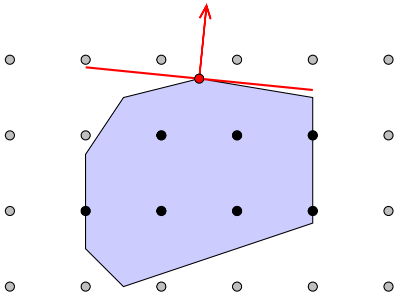

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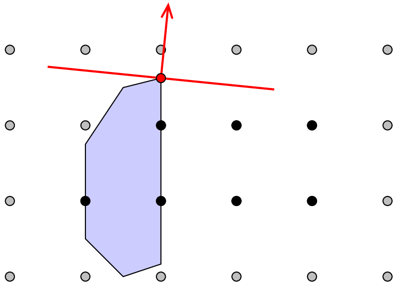

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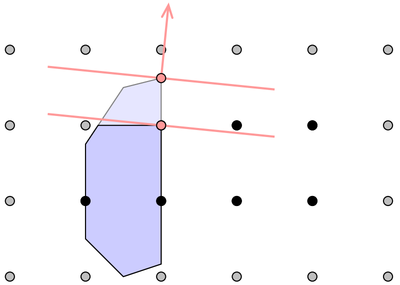

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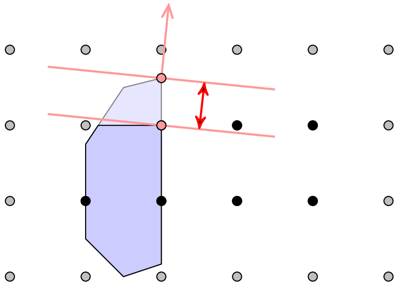

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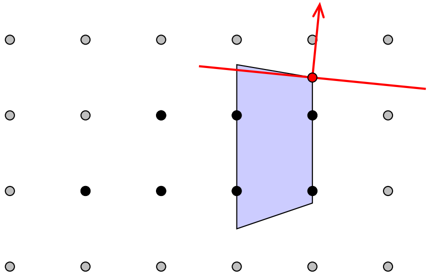

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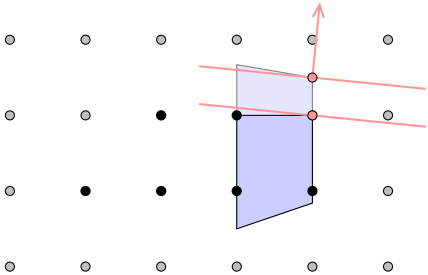

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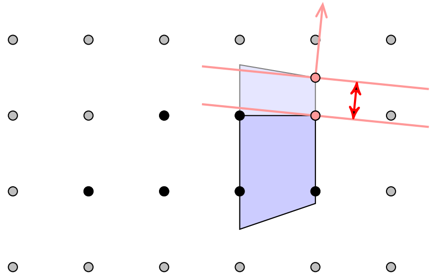

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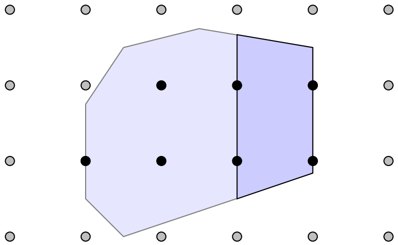

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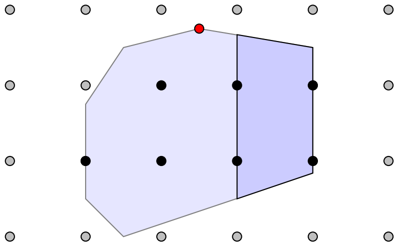

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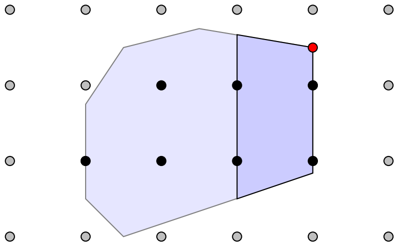

Used Information

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- Variables' pseudocosts:
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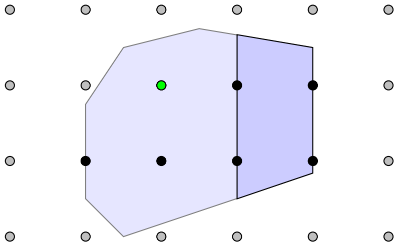

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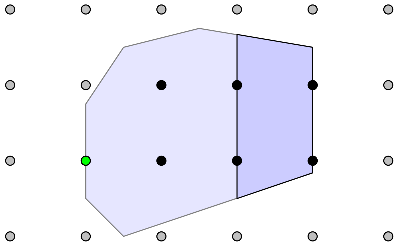

Approaches

- Rounding
- Diving: simulate DFS in the Branch-And-Bound-tree using some special branching rule
- Objective diving: manipulate objective function
- LNS: solve some sub-MIP
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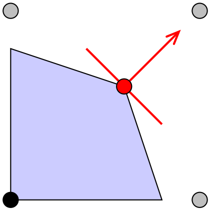
Implemented into SCIP

- 5 Rounding heuristics
- 8 Diving heuristics
- 3 Objective divers
- 4 LNS improvement heuristics

Ideas

- ▷ Simple Rounding always stays feasible,
- ▷ Rounding may violate constraints,
- ▷ Shifting may unfix integers.



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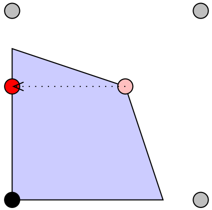

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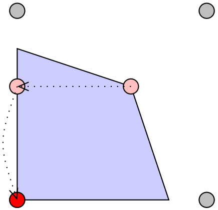

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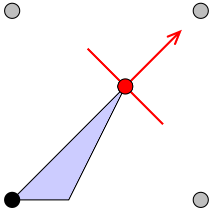

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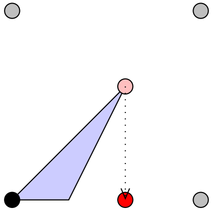

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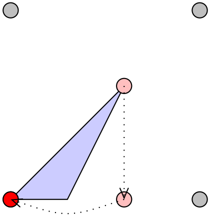

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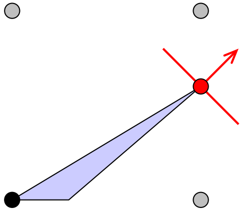

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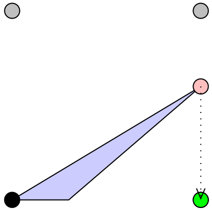

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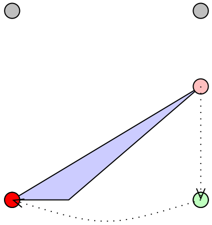


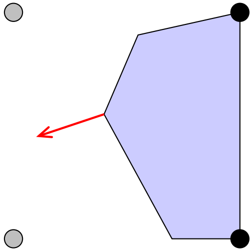
Idea: iteratively solve the LP and round a variable

Applied branching rules

- Fractional Diving: lowest fractionality
- Coefficient Diving: lowest locking number
- Linesearch Diving: highest increase since root
- Guided Diving: lowest difference to best known solution
- Pseudocost Diving: highest ratio of pseudocosts
- Vectorlength Diving: lowest ratio of objective change and number of rows containing the variable

Algorithm

1. Solve LP;
2. Round LP optimum;
3. If feasible:
4. Stop!
5. Else:
6. Change Objective;
7. Go to 1;



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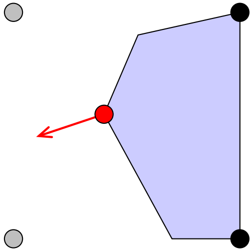

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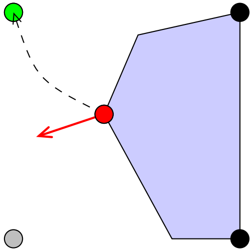

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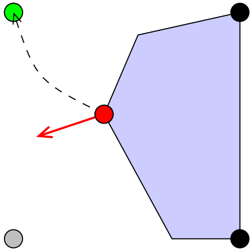

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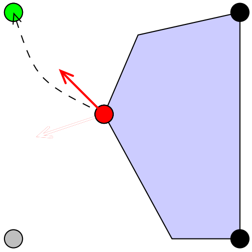

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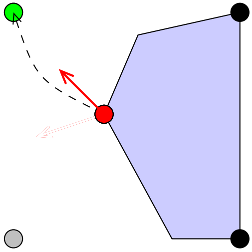

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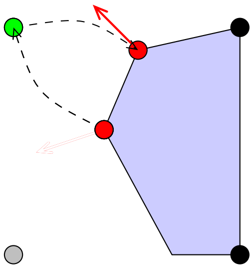

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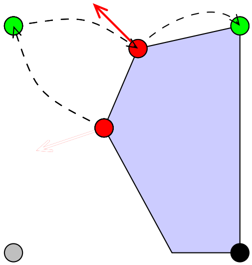

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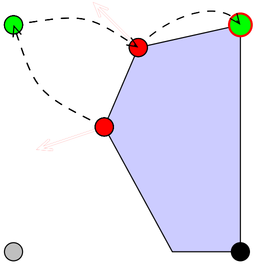

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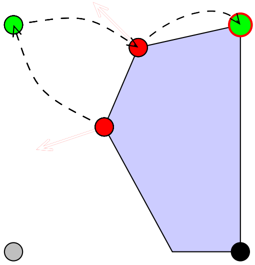

Improvements

- Objective $c^T x$ regarded at each step
- Algorithm able to resolve from cycling
- Quality of solutions much better

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- Algorithm able to resolve from cycling
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Results

- Finds a solution for 74% of the test instances
- On average 5.5 seconds running time
- Optimality gap decreased from 107% to 38%

Algorithm

1. $\bar{x} \leftarrow$ LP optimum;
2. Fix all integral variables:
   $x_i := \bar{x}_i \ \ \forall i : \bar{x}_i \in \mathbb{Z}$;
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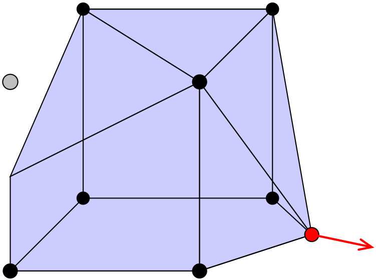

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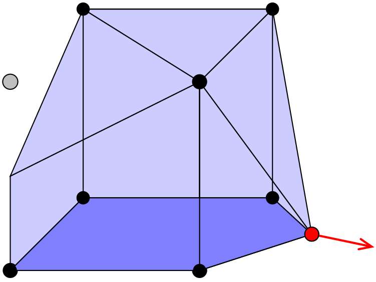

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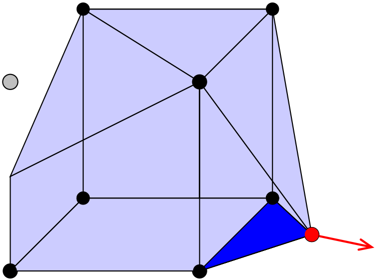

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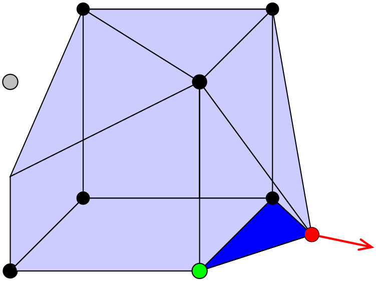

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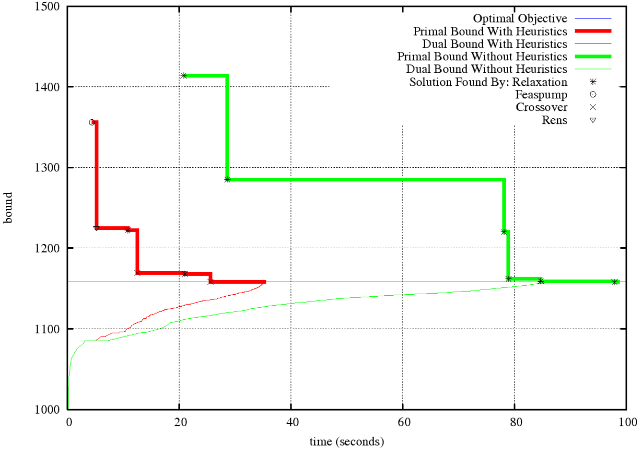
Results

- Approx. $\frac{2}{3}$ of the test instances are roundable
- Rens finds optimum for 20%!
- Dominates all other rounding heuristics



Instanz `aflow30a`: performance of SCIP with and without heuristics

Results

- Coordination important
- Positive side effects
- Improvement of overall performance
- SCIP with heuristics twice as fast

The MIP-Solving-Framework SCIP

Timo Berthold

Zuse Institut Berlin

DFG Research Center MATHEON

Mathematics for key technologies

Berlin, 23.05.2007