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Exercise sheet 4

Deadline: Thursday, May 24th, 2007, 08:30 h in MA-313

Exercise 1: **4 points**

Prove the following stronger version of the lemma for the component analysis separation method introduced in the exercises.

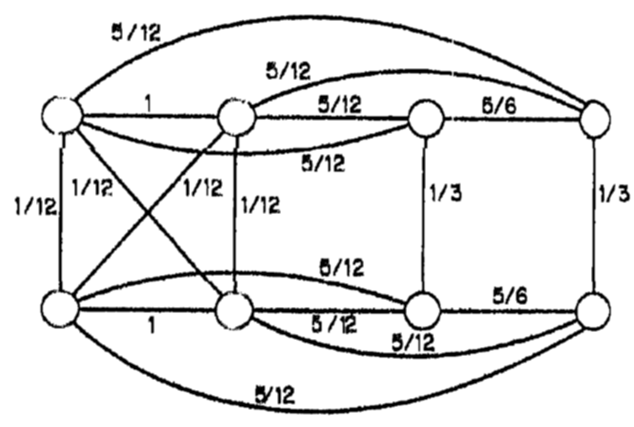
Lemma: Suppose $\bar{x}$ satisfies

$$\begin{aligned} x(\delta(v)) &= 2, \quad \forall v \in V \\ 0 \le x_e &\le 1 \quad \forall e \in E_n. \end{aligned}$$

Then the vertex set of every component of $G(w)$ defines a violated subtour elimination constraint if $G(w)$ is disconnected, where $w_{ij} = \max\{0, \bar{x} - 7/n^2\}$.

Exercise 2: **4 points**

a) Apply the shrinking heuristic to the following graph $G(\bar{x})$:



b) Find a violated subtour elimination constraint in the reduced graph.

Exercise 3: **4 points**

Prove the shrinking lemma:

Lemma: Suppose $G(\bar{x})$ is such that $\bar{x}_e = 1$. Let $G(\bar{x}') = (V', E')$ denote the graph after shrinking edge e.
There exists $W \subseteq V$, $W \neq \{i, j\}$, such that $\bar{x}(E(W)) > |W| - 1$ iff there exists $W' \subseteq V'$ s. t. $\bar{x}'(E(W')) > |W'| - 1$ when e is shrunk.

Exercise 4: **4 points**

Let $G = (V, E)$ be a graph.

a) The matching polytope Match(G) of G is given by

$$\begin{aligned} x_e \geq 0, &\quad \forall e \in E \\ x(\delta(i)) \leq 1, &\quad \forall i \in V. \end{aligned}$$

Show that the odd set inequalities $x(E(W)) \leq (|W| - 1)/2$ for $W \subseteq V$, $|W|$ odd, are in $e^1(\text{Match}(G))$.

b) The stable set polytope Stab(G) of G is given by

$$\begin{aligned} x_i \geq 0, &\quad \forall i \in V \\ x_i + x_j \leq 1, &\quad \forall ij \in E. \end{aligned}$$

Show that the odd cycle inequalities $x(E(C)) \leq (|C| - 1)/2$ for an odd cycle C in G are in $e^1(\text{Stab}(G))$.