

Cutting Planes in SCIP

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Berlin, 6th June 2007

- 1 Cutting Planes in SCIP
- 2 Cutting Planes for the 0-1 Knapsack Problem
 - 2.1 Cover Cuts
 - 2.2 Lifted Minimal Cover Cuts
- 3 Computational Results

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General cutting planes

- ▷ Complemented mixed integer rounding cuts
- ▷ Gomory mixed integer cuts
- ▷ Strong Chvátal-Gomory cuts
- ▷ Implied bound cuts

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Cutting planes for special problems

- ▷ 0-1 knapsack problem
- ▷ 0-1 single node flow problem
- ▷ Stable set problem

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I want to solve general MIPs!

Why do I care about cutting planes for special problems?

General Cutting Plane Method

$$\min\{\mathbf{c}^T \mathbf{x} : \mathbf{x} \in X^{MIP}\}$$

$$\min\{\mathbf{c}^T \mathbf{x} : \mathbf{x} \in X^{LP}\}$$

$$X^{MIP} := \{\mathbf{x} \in \mathbb{Z}^n \times \mathbb{R}^m : \mathbf{A}\mathbf{x} \leq \mathbf{b}\}$$

$$X^{LP} := \{\mathbf{x} \in \mathbb{R}^n \times \mathbb{R}^m : \mathbf{A}\mathbf{x} \leq \mathbf{b}\}$$

General Cutting Plane Method

$$\min\{c^T x : x \in X^{MIP}\}$$

$$X^{MIP} := \{x \in \mathbb{Z}^n \times \mathbb{R}^m : Ax \leq b\}$$

$$\min\{c^T x : x \in X^{LP}\}$$

$$X^{LP} := \{x \in \mathbb{R}^n \times \mathbb{R}^m : Ax \leq b\}$$

Observation

▷ If the data are rational, then

▶ $\text{conv}(X^{MIP})$ is a rational polyhedron

▶ we can formulate the MIP as $\min\{c^T x : x \in \text{conv}(X^{MIP})\}$

LP

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Problem (in general)

- ▶ Complete linear description of $\text{conv}(X^{MIP})$?
- ▶ Number of constraints needed to describe $\text{conv}(X^{MIP})$ is extremely large

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Idea

- ▶ Construct a polyhedron Q with
 - ▶ $\text{conv}(X^{MIP}) \subseteq Q \subseteq X^{LP}$
 - ▶ $\min\{c^T x : x \in \text{conv}(X^{MIP})\} = \min\{c^T x : x \in Q\}$

↪ Start with X^{LP} and add **inequalities which are valid for X^{MIP}** (but violated by the current LP solution) to X^{LP}

- ▶ Inequalities valid for a relaxation of X^{MIP} are valid for X^{MIP}
- ▶ Generating valid inequalities for a relaxation is often easier
- ▶ The intersection of the relaxations should be a good approximation of X^{MIP}

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Relaxations of X^{MIP}

1. Linear combinations of constraints defining X^{MIP}
(row of the simplex tab., single constraint)
2. Other information
 - ▶ Logical implications between binary variables
(conflict graph)
 - ▶ Logical implications between a binary and a real variable

General cutting planes

- ▶ Complemented mixed integer rounding cuts (Linear comb.)
- ▶ Gomory mixed integer cuts (Row of the simplex tab.)
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- ▶ Implied bound cuts (Logical impl.)

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Cutting planes for special problems

- ▶ 0-1 knapsack problem (Single constraint)
- ▶ 0-1 single node flow problem (Linear comb. and bounds)
- ▶ Stable set problem (Conflict graph)

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$\text{conv}(X^{BK})$

$$X^{BK} := \{x \in \{0, 1\}^n : \sum_{j \in N} a_j x_j \leq a_0\}$$

- ▷ $N = \{1, \dots, n\}$
- ▷ a_0 and a_j are integers for all $j \in N$
- ▷ $a_j > 0$ for all $j \in N$
(since binary variables can be complemented)
- ▷ $a_j \leq a_0$ for all $j \in N$
(since $a_j > a_0$ implies $x_j = 0$)

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Definition (Cover)

$$C \subseteq N$$

$$\triangleright \sum_{j \in C} a_j > a_0$$

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Theorem

If $C \subseteq N$ is a cover for X^{BK} , then the **cover inequality**

$$\sum_{j \in C} x_j \leq |C| - 1$$

is valid for X^{BK} .

Let $x^* \in [0, 1]^n \setminus \{0, 1\}^n$ be a fractional vector with $\sum_{j \in N} a_j x_j^* \leq a_0$.

Find $C \subseteq N$ with $\sum_{j \in C} a_j > a_0$ such that

$$\sum_{j \in C} x_j^* > |C| - 1,$$

or show that no inequality in the **class of cover inequalities** is violated by x^* .

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or show that no inequality in the **class of cover inequalities** is violated by x^* .

The separation problem can be formulated as a 0-1 KP.

Separation Problem as 0-1 KP

For $C \subseteq N$, we introduce the characteristic vector $z \in \{0, 1\}^n$.

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$$\sum_{j \in C} a_j > a_0 \Leftrightarrow \sum_{j \in N} a_j z_j > a_0$$

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Cover

$$\begin{aligned} \sum_{j \in C} a_j > a_0 &\Leftrightarrow \sum_{j \in N} a_j z_j > a_0 \\ &\Leftrightarrow \sum_{j \in N} a_j z_j \geq a_0 + 1 \end{aligned}$$

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Violated cover inequality

$$\sum_{j \in C} x_j^* > |C| - 1$$

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Violated cover inequality

$$\begin{aligned}\sum_{j \in C} x_j^* > |C| - 1 &\Leftrightarrow \sum_{j \in N} x_j^* z_j > \sum_{j \in N} z_j - 1 \\ &\Leftrightarrow \sum_{j \in N} (1 - x_j^*) z_j < 1\end{aligned}$$

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x^* satisfies all cover inequalities

$$\Leftrightarrow \forall z \in \{0, 1\}^n \text{ with } \sum_{j \in N} a_j z_j \geq a_0 + 1 : \sum_{j \in N} (1 - x_j^*) z_j \geq 1$$

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$$\Leftrightarrow \min \left\{ \sum_{j \in N} (1 - x_j^*) z_j : \sum_{j \in N} a_j z_j \geq a_0 + 1, \right. \\ \left. z \in \{0, 1\}^n \right\} \geq 1$$

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$$\Leftrightarrow \max \left\{ \sum_{j \in N} (1 - x_j^*) \bar{z}_j : \sum_{j \in N} a_j \bar{z}_j \leq \sum_{j \in N} a_j - (a_0 + 1), \right. \\ \left. \bar{z} \in \{0, 1\}^n \right\} \geq 1 - \sum_{j \in N} (1 - x_j^*)$$

Input : $c \in \mathbb{Q}_+^n$, $a \in \mathbb{Q}_+^n \setminus \{0\}$, and $b \in \mathbb{Q}_+$

Output: Feasible solution of $\max\{c^T x : a^T x \leq b, x \in \{0, 1\}^n\}$

```

1 Sort the indices such that  $\frac{c_1}{a_1} \geq \dots \geq \frac{c_n}{a_n}$ 
2  $\bar{a} \leftarrow 0$ 
3 for  $j \leftarrow 1$  to  $n$  do
4     if  $\bar{a} + a_j \leq b$  then
5          $x_j \leftarrow 1$ 
6          $\bar{a} \leftarrow \bar{a} + a_j$ 
7     else
8         while  $j \leq n$  do
9              $x_j \leftarrow 0$ 
10             $j \leftarrow j + 1$ 
11 return  $x$ 
    
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Solves the LP relaxation and rounds down the solution.

Input : $c \in \mathbb{Q}_+^n$, $a \in \mathbb{Q}_+^n \setminus \{0\}$, and $b \in \mathbb{Q}_+$

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Time complexity: $O(n \log n)$

Exact Algorithm for the 0-1 KP

Input : $c \in \mathbb{Q}_+^n$, $a \in \mathbb{Z}_+^n \setminus \{0\}$, and $b \in \mathbb{Z}_+$

Output: Optimal solution of $\max\{c^T x : a^T x \leq b, x \in \{0, 1\}^n\}$

Algorithm uses dynamic programming

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Output: Optimal solution of $\max\{c^T x : a^T x \leq b, x \in \{0, 1\}^n\}$

Algorithm uses dynamic programming

Time complexity: $O(nb)$

- ▶ A separator for cover cuts has only a limited effect on the overall performance of SCIP
- ▶ It seems to be important to separate strong cutting planes (facets or at least faces of reasonably high dimension)

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Can we strengthen the cover inequalities?

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Class of Minimal Cover Inequalities

Definition (Minimal cover)

$$C \subseteq N$$

$$\triangleright \sum_{j \in C} a_j > a_0$$

$$\triangleright \sum_{j \in C \setminus \{i\}} a_j \leq a_0 \text{ for all } i \in C$$

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Theorem

If $C \subseteq N$ is a minimal cover for X^{BK} , then the **minimal cover inequality**

$$\sum_{j \in C} x_j \leq |C| - 1$$

defines a facet of

$$\text{conv}(X^{BK} \cap \{x \in \{0, 1\}^n : x_j = 0 \text{ for all } j \in N \setminus C \}).$$

- ▷ (j_1, \dots, j_t) lifting sequence of the variables in $N \setminus C$
- ▷ $X^i := X^{BK} \cap \{\mathbf{x} \in \{0, 1\}^n : x_{j_{i+1}} = \dots = x_{j_t} = 0\}$

Sequential Up-Lifting

- ▷ (j_1, \dots, j_t) lifting sequence of the variables in $N \setminus C$
- ▷ $X^i := X^{BK} \cap \{x \in \{0, 1\}^n : x_{j_{i+1}} = \dots = x_{j_t} = 0\}$

$$\sum_{j \in C} x_j \leq |C| - 1 \quad \text{valid for } X^0$$

$$\sum_{j \in C} x_j + \alpha_{j_1} x_{j_1} \leq |C| - 1 \quad \text{valid for } X^1$$

⋮

$$\sum_{j \in C} x_j + \sum_{k=1}^t \alpha_{j_k} x_{j_k} \leq |C| - 1 \quad \text{valid for } X^t = X^{BK}$$

- ▷ (j_1, \dots, j_t) lifting sequence of the variables in $N \setminus C$
- ▷ $X^i := X^{BK} \cap \{x \in \{0, 1\}^n : x_{j_{i+1}} = \dots = x_{j_t} = 0\}$

Theorem

For each $i = 1, \dots, t$, consider the 0-1 knapsack problem

$$z_{j_i} = \max \left\{ \sum_{j \in C} x_j + \sum_{k=1}^{i-1} \alpha_{j_k} x_{j_k} : \sum_{j \in C} a_j x_j + \sum_{k=1}^{i-1} a_{j_k} x_{j_k} \leq a_0 - a_{j_i}, \right. \\ \left. x \in \{0, 1\}^{|C|+(i-1)} \right\}$$

and let $\alpha_{j_i} = (|C| - 1) - z_{j_i}$. Then for each $i = 1, \dots, t$,

$$\sum_{j \in C} x_j + \sum_{k=1}^i \alpha_{j_k} x_{j_k} \leq |C| - 1$$

defines a facet of $\text{conv}(X^i)$.

- ▷ (j_1, \dots, j_t) lifting sequence of the variables in $N \setminus C$
- ▷ $X^i := X^{BK} \cap \{\mathbf{x} \in \{0, 1\}^n : x_{j_{i+1}} = \dots = x_{j_t} = 0\}$

Different lifting sequences may lead to different inequalities!

Computing the Lifting Coefficients

- ▶ For each $i = 1, \dots, t$, solve the 0-1 KP
 - ▶ approximately ($O(n \log n)$)
 - ▶ exactly ($O(nb)$)
- ▶ Zemel: Exact algo to calculate all lifting coefficients ($O(n^2)$)
 - ▶ Uses dynamic programming to solve a reformulation of the 0-1 KPs
(role of the objective function and the constraint is reversed)

- ▶ Using sequential up-lifting to strengthen minimal cover cuts improves the performance of SCIP

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But, a separator which uses up- and down-lifting performs even better!

Theorem

If $C \subseteq N$ is a minimal cover for X^{BK} , then the **minimal cover inequality**

$$\sum_{j \in C} x_j \leq |C| - 1$$

defines a facet of

$$\text{conv}(X^{BK} \cap \{x \in \{0, 1\}^n : x_j = 0 \text{ for all } j \in N \setminus C \}).$$

\rightsquigarrow Up-lifting: variables in $N \setminus C$

Theorem

If $C \subseteq N$ is a minimal cover for X^{BK} and (C_1, C_2) is any partition of C with $C_1 \neq \emptyset$, then inequality

$$\sum_{j \in C_1} x_j \leq |C_1| - 1$$

defines a facet of

$$\text{conv}(X^{BK} \cap \{x \in \{0, 1\}^n : x_j = 0 \text{ for all } j \in N \setminus C, \\ x_j = 1 \text{ for all } j \in C_2 \}).$$

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↪ Up-lifting: variables in $N \setminus C$

↪ Down-lifting: variables in C_2

- ▷ Similar theorem as for sequential up-lifting
- ▷ Extension of Zemel's up-lifting procedure can be used
- ▷ In SCIP, the separation problem for the **class of lifted minimal cover inequalities using sequential up- and down-lifting** is solved heuristically

Outline of the Separation Algorithm

Step 1 (Cover)

- ▷ Determine a cover C for X^{BK}
(separation problem for the class of cover inequalities)

Step 2 (Minimal cover and partition)

- ▷ Make the cover minimal by removing vars from C
- ▷ Find a partition (C_1, C_2) of C with $C_1 \neq \emptyset$

Step 3 (Lifting)

- ▷ Determine a lifting sequence of the variables in $N \setminus C_1$
- ▷ Lift the inequality $\sum_{j \in C_1} x_j \leq |C_1| - 1$ using sequential up- and down-lifting

Step 1 (Cover)

- ▷ Which algorithm do we use to find the cover?
 - ▶ Fixing of variables
 - ▶ Modification of the separation problem (Gu et al. (1998))
 - ▶ Solving the separation problem exactly or approximately

Step 2 (Minimal cover and partition)

- ▷ In which order do we remove the variables?
- ▷ Which partition of the minimal cover do we use?

Step 3 (Lifting)

- ▷ Which lifting sequence do we use?
- ▷ Which algorithm do we use to solve the knapsack problems that occur in the sequential lifting procedure?

Step 1 (Cover)

- ▷ Fix all vars with $x_j^* = 0$ to zero and all vars with $x_j^* = 1$ to one
- ▷ Use the modification of the separation problem
- ▷ Solve the modified separation problem approximately

Step 2 (Minimal cover and partition)

- ▷ Nondecreasing order of x_j^* and nondecreasing order of a_j
- ▷ $C_2 := \{j \in C : x_j^* = 1\}$ ($|C_1| = 1$: change the partition)

Step 3 (Lifting)

- ▷ $\{j \in N \setminus C : x_j^* > 0\}$, C_2 , and then $\{j \in N \setminus C : x_j^* = 0\}$
(nonincreasing order of a_j)
- ▷ Use an extension of Zemel's procedure

	Gap Closed % (Geom. Mean)		Sepa Time sec (Total)	
	Value	Δ	Value	Δ
Default algorithm	16.31	0.00	1355.9	0.0
Resulting algorithm	16.36	0.05	7.4	-1348.5

	Gap Closed % (Geom. Mean)		Sepa Time sec (Total)	
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Default algorithm	16.31	0.00	1355.9	0.0
Cover – 1. modification	15.61	-0.70	7.6	-1348.3
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- ▷ Determination of the cover
 - ▶ Solve the separation problem approximately

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Cover – 1. modification	15.61	-0.70	7.6	-1348.3
Cover – 2. modification	16.42	0.11	7.1	-1348.8
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- ▷ Determination of the cover
 - ▶ Solve the separation problem approximately
 - ▶ Modification of the separation problem

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Comparison with other MIP solvers

- ▶ Are the cutting plane separators implemented in SCIP competitive to the ones included in other MIP solvers?

MIP solvers

- ▶ SCIP (CPLEX as LP solver)
- ▶ CPLEX
- ▶ COIN-OR Branch and Cut solver (COIN-OR LP solver)

MIP solvers

- ▷ SCIP (CPLEX as LP solver)
- ▷ CPLEX
- ▷ COIN-OR Branch and Cut solver (COIN-OR LP solver)

Settings

- ▷ One cutting plane separator
- ▷ Isolated application
- ▷ Presolving disabled (used presolved instances obtained by the presolving routines of CPLEX)

MIP solvers

- ▷ SCIP (CPLEX as LP solver)
- ▷ CPLEX
- ▷ COIN-OR Branch and Cut solver (COIN-OR LP solver)

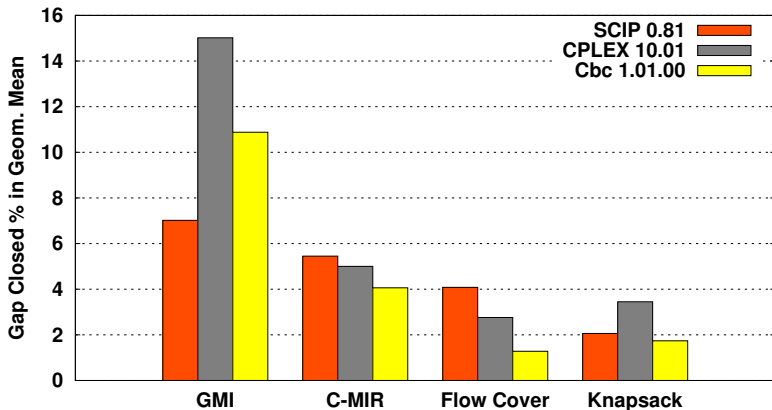
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- ▷ One cutting plane separator
- ▷ Isolated application
- ▷ Presolving disabled (used presolved instances obtained by the presolving routines of CPLEX)

Test set

- ▷ 134 instances (MIPLIB 2003, MIPLIB 3.0, and MIP collection of Mittelmann)

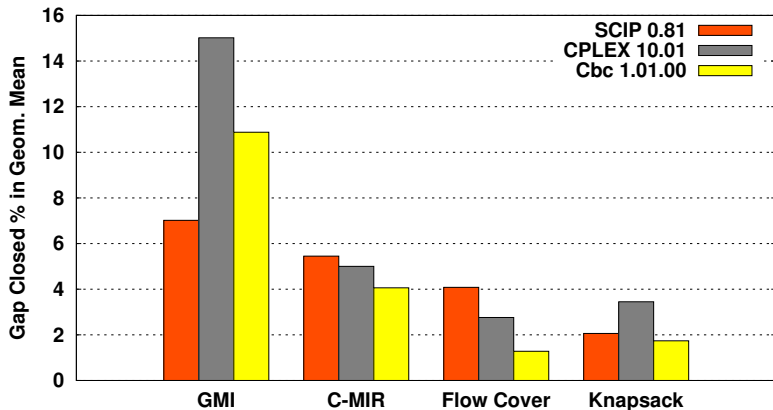
Computational Results



Performance measure

$$\text{Gap Closed \%} \quad \left(100 \cdot \frac{db - z_{LP}}{z_{MIP} - z_{LP}} \right)$$

Computational Results



Knapsack

CPLEX: Time in total $\approx 9,400$ sec

SCIP and CBC: Time in total ≈ 600 sec

Impact on the overall performance of SCIP

- ▷ How strong is the impact of the individual cutting plane separators on the overall performance of SCIP?

Impact of the individual cutting plane separators when they are used

as the only separators in
SCIP

1. Started with running
SCIP **without any
separators**
2. Compared the
performance with the
one of SCIP when we
enabled one separator

in connection with all other
separators of SCIP

1. Started with running
SCIP **with all
separators**
2. Compared the
performance with the
one of SCIP when we
disabled one separator

Performance measures

- ▷ Nodes
- ▷ Time
- ▷ Gap %

Performance measures

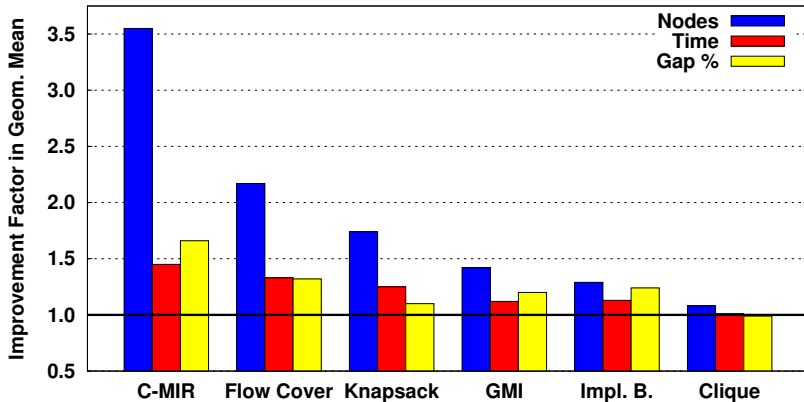
- ▷ Nodes
- ▷ Time
- ▷ Gap %

Improvement factor for each performance measure

$$\frac{\text{Value for SCIP run without any separators}}{\text{Value for SCIP run with one separator enabled}}$$

- ↪ Factor by which enabling the separator improves the overall performance (Factor > 1?)

Enabling – Computational Results



Performance measures

- ▷ Nodes
- ▷ Time
- ▷ Gap %

Performance measures

- ▷ Nodes
- ▷ Time
- ▷ Gap %

Degradation factor for each performance measure

$$\frac{\text{Value for SCIP run with one separator disabled}}{\text{Value for SCIP run with all separators}}$$

- ↪ Factor by which disabling the separator degrades the overall performance (Factor > 1?)

Disabling – Computational Results

