MAXIMAL REGULARITY AND APPLICATIONS TO PDES

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1. Description of the problem

The purpose of this series of lectures is to give a flavor of the concept of maximal regularity. In the last ten-fifteen years, a lot of progress has been made on this subject. The problem of parabolic maximal L^p-regularity can be stated as follows.

> Let A be an (unbounded) operator on a Banach space X, with domain $D(A)$. Let $p \in]1, \infty[$. Does there exist a constant $C > 0$ such that for all $f \in L^p(0, \infty; X)$, there exists a unique $u \in L^p(0, \infty; D(A)) \cap L^p_1(0, \infty; X)$ solution of $u' + Au = f$ and $u(0) = 0$ verifying
>
> $$\|u'\|_{L^p(0,\infty;X)} + \|Au\|_{L^p(0,\infty;X)} \leq \|f\|_{L^p(0,\infty;X)}?$$

Applications to partial differential equations, such as incompressible Navier-Stokes equations, will be given.

2. Table of contents

The outline of the lectures is, up to minor changes,

(1) Statement of the problem
(2) Basic examples
(3) Application to Navier-Stokes equations
(4) Non-autonomous maximal regularity
(5) Applications to quasilinear parabolic equations

3. Prerequisites

It is better, but not mandatory, if the audience has already an idea of basic semigroup theory, at least applied to basic examples such as the heat equation.