Technische Universität Berlin **Sommersemester 2011**
Fakultät II – Mathematik und Naturwissenschaften
Institut für Mathematik
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Numerische Mathematik II/ Numerical Analysis II
2. Assignment

Homework: HW2 (Due 04.05.2011)

1. Construct the adjoint of Heun's method.
2. Show that the implicit trapezoidal rule
$$u_{i+1} = u_i + \frac{h}{2}(f(t_i, u_i) + f(t_{i+1}, u_{i+1}))$$
is symmetric.
3. Show that the adjoint method has exactly the same asymptotic expansion for the global error as the original method, with h replaced by $-h$.
4. Show that the symmetric method of order p possess an h^2-expansion of the global error of the form
$$e(t,h) = y(t) - u_h(t) = e_{2q}(t)h^{2q} + e_{2q+2}(t)h^{2q+2} + \dots$$
with $e_{2j}(t_0) = 0$ and $q = \lceil \frac{p}{2} \rceil$.

Programming assignment: PA2 (Due 09.05.2011/10.05.2011)

1. Write a program that solves the ODE $y'(t) = f(t,y)$, $y(t_0) = y_0$ using Gragg's method without and with extrapolation (GBS-method) on the interval $[t_0, t_0 + a]$.
   (a) For Gragg's method without extrapolation compute $S_0 = y_0, S_1, S_2, \dots, S_N$ where
$$\begin{aligned} u_0 &= y_0 \\ u_1 &= u_0 + hf(t_0, y_0) \\ u_{i+1} &= u_{i-1} + 2hf(t_i, u_i), \quad i = 1, 2, \dots, N \\ S_i &= \frac{1}{4}(u_{i-1} + 2u_i + u_{i+1}), \quad i = 1, 2, \dots, N. \end{aligned}$$
   Compare Gragg's method with Runge-Kutta method from the previous programming assignment for $N(i) = 10, 20, 40, 80, 160, 320, 640, 1280, 2560, 5120$.
   (b) For Gragg's method with extrapolation use the Neville-Aitken method introduced in the lecture with $H = \frac{a}{N(1)}$. Compare the results for $h_l = \frac{H}{2^l}$, $l = 0, 1, 2, \dots$ and $h_l = \frac{H}{2l}$, $l = 1, 2, 3, \dots$, where l is the number of extrapolation steps. Extrapolate the solution in points $t_0 + H$, $t_0 + 2H$, $t_0 + 3H$, ...

   Test your program for the problems from Exercise 1 Assignment 1 with $a = 1$. For steps $t_0, t_0 + \frac{1}{5}a, t_0 + \frac{2}{5}a, \dots$ plot the numerical solution against the exact solution and the corresponding error $\mathsf{e}_i = \|y(t_i) - \mathsf{u}_i\|_\infty$. Summarize the numerical results in a table.

Hint:

```
[h,t,u] = gragg(f,t0,y0,N,a,l)
```

where `t0` $= t_0$ is the beginning of the interval, `y0` $= y_0$ the initial value, `N` $= N$ total number of steps and `a` $= a$ the interval length, `l` the number of extrapolation points (`l` $= 0$ no extrapolation). The output are `h` $= h = a/N$ the step size, `u` $= \mathbf{u} = [u_1, \ldots, u_n]$ the numerical approximation of $y(t)$ at points `t` $= [t_0, \ldots, t_N]$.

For Gragg's method with extrapolation your program may call subroutines `graggstep` and `extrapol`. Subroutine `graggstep` may determine the approximate solutions $u_{h_i}(t_0 + H)$ using local step sizes h_i.