**Technische Universität Berlin** **Sommersemester 2011**
**Fakultät II – Mathematik und Naturwissenschaften**
**Institut für Mathematik**
Prof. Dr. Christian Mehl, Agnieszka Miedlar Stand: May 13, 2011

# Numerische Mathematik II/ Numerical Analysis II
# 4. Assignment

**Exercise: EX4 (10-11.05.2011)**

1. Solve the initial value problem
$$y'(t) = t^2 + y^3, \quad y(1) = 0$$
in the interval [1, 1.6] using the predictor-corrector method $P(EC)^{m_0}$
$$\text{P:} \quad u_{j+2} = u_{j+1} + \tfrac{h}{2}(3f_{j+1} - f_j)$$
$$\text{C:} \quad u_{j+2} = u_{j+1} + \tfrac{h}{12}(5f_{j+2} + 8f_{j+1} - f_j)$$
with $h = 0.2$. Use second order Taylor series method with step-size $h = 0.2$ to calculate the starting values. Perform three corrector steps per iteration, i.e., $m_0 = 3$.

2. Determine the solutions for the following linear difference equations
   (a) $u_{j+2} + u_{j+1} - 2u_j = 0, \quad u_0 = 2, u_1 = -1,$
   (b) $u_{j+1} - u_j = j, \quad u_0 = 0.$

**Homework: HW4 (17-18.05.2011)**

1. Solve the initial value problem **(4 pts.)**
$$y'(t) = y + \sin y, \quad y(1) = 1$$
in the interval [1, 1.6] using the predictor-corrector method $P(EC)^{m_0}$
$$\text{P:} \quad u_{j+2} = u_{j+1} + \tfrac{h}{2}(3f_{j+1} - f_j)$$
$$\text{C:} \quad u_{j+1} = u_j + \tfrac{h}{2}(f_{j+1} + f_j)$$
with $h = 0.2$. Use second order Taylor series method with step-size $h = 0.2$ to calculate the starting values. Perform two corrector steps per iteration, i.e., $m_0 = 2$.

2. Determine the solutions for the following linear difference equations **(4 pts.)**
   (a) $u_{j+2} - 6u_{j+1} + 9u_j = 0,$
   (b) $u_{j+2} + 4u_j = 0, \quad u_0 = 0,$
   (c) $u_{j+2} - 4u_{j+1} - 5u_j = 1, \quad u_0 = 0, u_1 = 0,$
   (d) $u_{j+1} - u_j = 2^j, \quad u_0 = 0.$

3. Let the matrix $A$ be given as **(6 pts.)**
$$A = \begin{bmatrix} 0 & 1 & & \\ & \ddots & \ddots & \\ & & 0 & 1 \\ -\alpha_0 & \dots & -\alpha_{k-2} & -\alpha_{k-1} \end{bmatrix}.$$

(a) Show that the characteristic polynomial $\det(\lambda I - A)$ of matrix $A$ is

$$p(\lambda) = \lambda^k + \alpha_{k-1}\lambda^{k-1} + \ldots + \alpha_1\lambda + \alpha_0.$$

(b) Prove that $\dim \operatorname{Ker}(A - \lambda I) \leq 1$ for all $\lambda \in \mathbb{C}$.

4. Consider the inhomogeneous linear difference equation **(6 pts.)**

$$u_{j+k} + \alpha_{k-1}u_{j+k-1} + \ldots + \alpha_0 u_j = f_j, \quad (*)$$

with given $u_0, \ldots, u_{k-1}$, $u_i, f_i \in \mathbb{R}^n$.

(a) Show that $(*)$ can be transformed to a system of linear difference equations of order one of the form

$$\widehat{U}_{j+1} = \widehat{A}\widehat{U}_j + \widehat{F}_j.$$

How are $\widehat{U}_{j+1}$, $\widehat{A}$ and $\widehat{F}_j$ defined?

(b) Using (a) determine the general formula for the solution $\widehat{U}_{j+1}$ depending on $U_0$.

(c) Using (b) write the solution of the inhomogeneous linear difference equation

$$u_{j+2} - 2u_{j+1} - 3u_j = 1, \text{ with } u_0 = 0, \; u_1 = 1.$$