Technische Universität Berlin **Sommersemester 2011**
Fakultät II – Mathematik und Naturwissenschaften
Institut für Mathematik
Prof. Dr. Christian Mehl, Dr. Agnieszka Miedlar Stand: May 30, 2011

Numerische Mathematik II/ Numerical Analysis II
6. Assignment

Homework: HW6 (31.05 - 01.06.2011)

1. Find and sketch the regions of absolute stability of the given methods **(4 pts.)**
   (a) $u_{j+3} = u_{j+2} + \frac{h}{24}(9f_{j+3} + 19f_{j+2} - 5f_{j+1} + f_j)$
   (b) $u_{j+2} = u_{j+1} + \frac{h}{12}(5f_{j+2} + 8f_{j+1} - f_j)$
   (c) $u_{j+2} = u_j + 2hf_{j+1}$
   (d) $u_{j+1} = u_j + \frac{h}{2}(f_j + f_{j+1})$
2. Investigate the consistency, stability and convergence of the following methods **(4 pts.)**
   (a) $u_{j+2} = 4u_{j+1} - 3u_j - 2hf_j$
   (b) $u_{j+1} = u_j + hf_{j+1}$
   (c) $u_{j+2} = \frac{3}{2}u_{j+1} - \frac{1}{2}u_j + hf_{j+1}$
   (d) $u_{j+2} = 3u_{j+1} - 2u_j + h(f_{j+2} - 2f_{j+1})$
3. Consider the family of methods **(4 pts.)**
$$u_{j+1} = u_j + h(\theta f(t_j, u_j) + (1-\theta)f(t_{j+1}, u_{j+1})), \quad \theta \in [0,1].$$
   Determine the set of all θ, so that the associated method is A-stable.
4. Analogously to the BDF method from the lecture construct the two-step method which satisfies
$$P'(t_{j+1}) = f(t_{j+1}, u_{j+1}),$$
   where P is a polynomial such that
$$P(t_j) = u_j,\ P(t_{j+1}) = u_{j+1},\ P'(t_j) = f(t_j, u_j).$$ **(3 pts.)**

Programming Assignment: PA5 (7-8.06.2011) **(10 pts.)**
Write a program `diffsolve.m` which solves the boundary value problem

$$u''(t) + \sin(t)u'(t) + \cos(t)u(t) = 0 \quad \text{and} \quad u(-\frac{\pi}{2}) = u(\frac{3\pi}{2}) = 1,$$

by using difference scheme on the interval $\mathbb{I} = [-\frac{\pi}{2}, \frac{3\pi}{2}]$. Use the forward type 1st order difference quotient and the 2nd order central difference quotient.

Program structure:

The routine `diffsolve.m` should be of the following form

```
[t,u] = diffsolve(N).
```

For a given number of steps `N` the interval $\mathbb{I}$ will be approximated with the equidistant mesh $\mathbb{I}_h = [-\frac{\pi}{2} = t_0, t_1, \ldots, t_{N-1}, t_N = \frac{3\pi}{2}]$. The solution of the boundary-value problem in those mesh points will be approximated by the mesh function $u_j = u(t_j)$, $j = 0, 1, \ldots, N$. The resulting linear system may be solved with help of Matlab \. Plot the solution for $N = 2^j$, $j = 3, \ldots, 10$ and the error in logarithmic scale. The exact solution of the problem is given by $u(t) = e^{\cos(t)}$. Interpret your results.