Technische Universität Berlin **Sommersemester 2011**
Fakultät II – Mathematik und Naturwissenschaften
Institut für Mathematik
Prof. Dr. Christian Mehl, Dr. Agnieszka Miedlar Stand: June 1, 2011

Numerische Mathematik II/ Numerical Analysis II
7. Assignment

Homework: HW7 (7-8.06.2011)

1. Check the existence and uniqueness of the solution of the following boundary value problems

   (a) $y'' + 9y = 0, \quad y(0) = 1, \quad y'(\frac{\pi}{2}) = 3,$

   (b) $y'' + 2y' + 2y = 0, \quad y(0) - y'(0) = 1, \quad y'(\pi) - y(\pi) = e^{-\pi}.$

   (2 pts.)

2. Using the Gershgorin's Theorem

   (a) locate the eigenvalues of the matrix **(2 pts.)**

$$M = \begin{bmatrix} 5 & 0 & 0 & -1 \\ 1 & 0 & -1 & 1 \\ -1.5 & 1 & -2 & 1 \\ -1 & 1 & 3 & -3 \end{bmatrix}.$$

   Compare your results with those given by Matlab function `eig`.

   (b) show that matrix $A = [a_{i,j}] \in \mathbb{C}^{n\times n}$ is invertible if it is strictly diagonally dominant. Is this also true for the diagonally dominant matrices? (Proof or give a counter example).
   Note: Matrix $A = [a_{i,j}] \in \mathbb{C}^{n\times n}$ is

$$\text{diagonally dominant} \Leftrightarrow |a_{i,i}| \geq \sum_{j=1, j\neq i}^{n} |a_{i,j}|,$$

$$\text{strictly diagonally dominant} \Leftrightarrow |a_{i,i}| > \sum_{j=1, j\neq i}^{n} |a_{i,j}|.$$

   (3 pts.)

3. Let the following matrices have appropriate sizes. Show that **(4 pts.)**

   (a) $0 \leq B_1 \leq B_2 \wedge 0 \leq D_1 \leq D_2 \Rightarrow B_1 \cdot D_1 \leq B_2 \cdot D_2,$

   (b) $|B \cdot D| \leq |B| \cdot |D|.$

4. Discretize the following boundary value problems using **(4 pts.)**

   (a) the 1st and the 2nd order central DQ

$$y''(t) = -\frac{4}{t}y'(t) + \frac{2}{t^2}y(t) - \frac{2}{t^2}\log(t), \quad y(1) = -\frac{1}{2},\ y(2) = \log 2,\ t \in [1,2],$$

   (b) the 1st order forward DQ and 2nd order central DQ

$$y''(t) + 2ty'(t) + y(t) - t^2 = 0, \quad y(0) = 0,\ y(1) = 0$$

   and write the corresponding equations in the matrix form $\mathbf{Au} = \mathbf{b}$.

Programming Assignment: PA7 (21-22.06.2011) **(10 pts.)**

Write a program `shooting.m` which solves the boundary value problem $y''(t) = f(t, y(t), y'(t))$, $y(a) = ya$, $y(b) = yb$ using the single shooting method with Newton's method.

Program structure:

On the web page you will find the file `shooting.m`, the file `runme.m` and all necessary subprograms. Your task is to complete the `shooting.m` file. In the `runme.m` you will test your program for problems

1. $y'' = 1 + (y')^2,\ y(0) = 0, y(\frac{\pi}{4}) = 1,\ \mathbb{I} = [0, \frac{\pi}{4}]$ `(funct6)`
2. $y' = -\tan(\frac{1}{1.05-t})\frac{y}{(1.05-t)^2},\ y(0.98) = \cos(\frac{1}{(1.05-0.98)})$ `(funct9)`

Choose `tol`$= 10^{-2}, 10^{-4}, 10^{-6}$.
Submission: In addition to `shooting.m` file, please submit printouts of the files generated by `runme.m`.