Gragg's method as a one-step method

Define: $h^* = 2h$

$$t_k^* = t_0 + k \cdot h^*$$

$$v_0 = u_0$$

$$w_0 = u_0$$

$$v_k = u_{2k}$$

$$w_k = u_{2k+1} - h f(t_{2k}, u_{2k}) = \frac{1}{2}\left(u_{2k-1} + u_{2k+1}\right)$$

Then: $$\begin{bmatrix} v_{k+1} \\ w_{k+1} \end{bmatrix} = \begin{bmatrix} v_k \\ w_k \end{bmatrix} + h^* \begin{bmatrix} f\left(t_k^* + \frac{h^*}{2}, w_k + \frac{h^*}{2} f(t_k^*, v_k)\right) \\ \frac{1}{2}\left(f(t_k^* + h^*, v_{k+1}) + f(t_k^*, v_k)\right) \end{bmatrix}$$

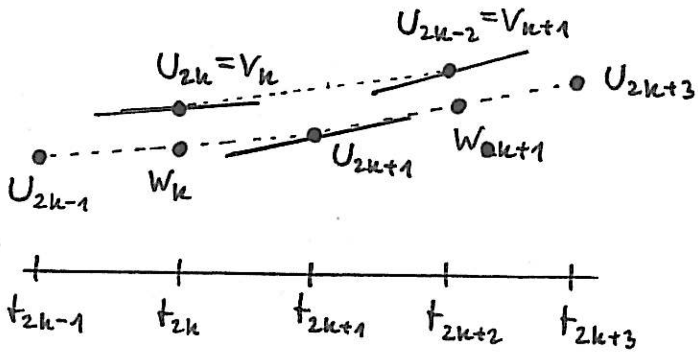
because

$$\begin{aligned} v_{k+1} = u_{2k+2} &= u_{2k} + 2h f(t_{2k+1}, u_{2k+1}) \\ &= v_k + h^* f\left(t_k^* + \frac{h^*}{2}, w_k + h f(t_{2k}, u_{2k})\right) \\ &= v_k + h^* f\left(t_k^* + \frac{h^*}{2}, w_k + \frac{h^*}{2} f(t_k^*, v_k)\right) \end{aligned}$$

and

$$\begin{aligned} w_{k+1} &= \frac{1}{2}\left(u_{2k+3} + u_{2k+1}\right) = \frac{1}{2}\left(u_{2k+1} + 2h f(t_{2k+2}, u_{2k+2}) + u_{2k-1} + 2h f(t_{2k}, u_{2k} \right. \\ &= w_k + \frac{1}{2}\left(h^* f(t_k^* + h^*, v_{k+1}) + h^* f(t_k^*, v_k)\right) \end{aligned}$$

The method is symmetric:



We obtain $S_h(t_0 + 2nh) = \frac{1}{2}(v_n + w_n)$