Technische Universität Berlin **Sommersemester 2012**
Fakultät II – Mathematik und Naturwissenschaften
Institut für Mathematik
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Systems and control theory

Series 2

Task 1: ⊖

1. Let $P \in \mathbb{C}[\lambda]^{p,q}$ have full row rank $p = \operatorname{rank}_{\mathbb{C}(\lambda)}(P)$. Use the Smith form to construct a (rational) right inverse, i.e., a matrix $Q \in \mathbb{C}(\lambda)^{q,p}$ such that $PQ = I$. Under which conditions can Q be chosen polynomial? What are the poles of Q?

2. Let $R \in \mathbb{C}(\lambda)^{p,q}$ be a rational matrix with full row rank $p = \operatorname{rank}_{\mathbb{C}(\lambda)}(R)$. Use the MacMillan form to construct a (rational) right inverse, i.e., a matrix $Q \in \mathbb{C}(\lambda)^{q,p}$ such that $RQ = I$. Under which conditions can Q be chosen polynomial? What are the poles and zeros of Q?

Task 2: ⊖
Assume that $\tilde{U}$ and $\tilde{V}$ are kernel and co-kernel spanning matrices. Further assume that $\hat{U}$ and $\hat{V}$ are kernel and co-kernel spanning matrices. Show that then also $\tilde{U}$ and $\hat{V}$ (as well as $\hat{U}$ and $\tilde{V}$) are kernel and co-kernel spanning matrices.
Remark: This shows that one can speak of kernel and co-kernel spanning matrices independently.

Task 3: ⊕
Let $P \in \mathbb{C}[\lambda]^{p,q}$. Use the Smith form to show that there exists a unimodular matrix $U \in \mathbb{C}[\lambda]^{p,p}$ such that $UP = \begin{bmatrix} \tilde{P} \\ 0 \end{bmatrix}$, where $\tilde{P}$ has full row rank.
Conclude that for every $P \in \mathbb{C}[\lambda]^{p,q}$ with $r := \operatorname{rank}_{\mathbb{C}(\lambda)}(P)$ there exists a $\tilde{P} \in \mathbb{C}[\lambda]^{r,q}$ such that $\mathcal{B}(P) = \mathcal{B}(\tilde{P})$, i.e., that one can theoretically restrict the attention to kernel representations which come from polynomial matrices with full row rank.

Task 4: ⊕
Show that $\mathcal{B}(P)$ is autonomous if and only if the following holds:

$$\text{for every } z \in \mathcal{B}(P) \text{ with } z(t) = 0 \text{ for } t \leq 0 \text{ we already have } z = 0.$$

Task 5: ⊕
Give the analog to Theorem 1.13 which talks about right primeness.

Task 6: ⊕

1. Show that kernel and co-kernel spanning matrices have full column rank.

2. For every $R \in \mathbb{C}(\lambda)^{p,q}$ there exists a polynomial kernel matrix which is (right) prime.

3. For every $R \in \mathbb{C}(\lambda)^{p,q}$ there exists a polynomial co-kernel matrix which is (right) prime.

Task 7: ⊚
Show that a latent variable description defines a linear time-invariant dynamical system.

Task 8: $\circledcirc$

Show that for a square polynomial matrix $P \in \mathbb{C}[\lambda]^{p,p}$ the following are equivalent:

1.) P is prime
2.) P is left prime
3.) P is right prime
4.) P is unimodular
5.) $\det P$ is prime (i.e. a nonzero constant)
6.) $\det P$ is unimodular
7.) P has no zeros, i.e., $\mathfrak{Z}(P) = \emptyset$, and $\operatorname{rank}_{\mathbb{C}(\lambda)}(P) = p$

Task 9: $\oplus$

Let $P \in \mathbb{C}[\lambda]^{p,q}$ and let $I \subset \mathbb{R}$ be some bounded interval. Let $z \in \mathcal{C}^q_\infty$ vanish identically in I, i.e., let $z(t) = 0$ for $t \in I$. Show that then $P\left(\frac{d}{dt}\right) z(t) = 0$ for $t \in I$.

Task 10: $\oplus$

Consider the example from the last series:

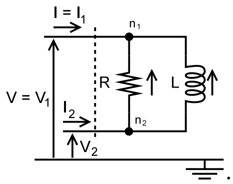

Figure 1: A simple RL circuit

With the definitions

$$P(\lambda) := \begin{bmatrix} 1 & 1 & 1 & 0 \\ -R & 0 & 0 & 1 \\ 0 & \lambda L & 0 & -1 \end{bmatrix} \qquad \text{and} \qquad z := \begin{bmatrix} I_R \\ I_L \\ I \\ V \end{bmatrix} \tag{1}$$

the system was given by $\mathcal{B}(P)$.

1. Consider I_R and I_L to be latent variables. Conduct elimination of latent variables to obtain $\tilde{P} \in \mathbb{C}[\lambda]^{1,2}$ which describes the manifest behavior. Use the constructive proof of Theorem 1.16 to do so.
2. Is P prime? Is $\tilde{P}$ prime?
3. Is $\mathcal{B}(P)$ autonomous? Connect a capacitor with capacity $C > 0$ along the two external wires, i.e., add the addtional equation $C\dot{V} = I$ to the system given by the matrix in (1). Is the resulting system autonomous?

Task 11: $\ominus$

Let $P \in \mathbb{C}[\lambda]^{p,q}$ and let $P = S \begin{bmatrix} D & 0 \\ 0 & 0 \end{bmatrix} T$ be the MacMillan form. Obtain the echelon form from the MacMillan form.

Task 12: $\circledcirc$

1. Let (P, Q) be a partition of $R \in \mathbb{C}[\lambda]^{p,q}$. Prove the existence of a unimodular U such that
$$U \begin{bmatrix} P & Q \end{bmatrix} = \begin{bmatrix} P_1 & Q_1 \\ 0 & Q_2 \\ 0 & 0 \end{bmatrix},$$
where P_1 and Q_1 have full row rank.
2. Conclude that if R has full column rank, then P_1 and Q_1 are invertible.