Technische Universität Berlin **Sommersemester 2012**
Fakultät II – Mathematik und Naturwissenschaften
Institut für Mathematik
Tobias Brüll

Systems and control theory

Series 3

Task 1:

1. Give the Smith form for every block in the Kronecker canonical from separately. ⊕
2. Give a prime polynomial kernel spanning matrix for every block in the Kronecker canonical form separately. ⊕
3. Give the behavior for every block in the Kronecker canonical form separately. ⊕
4. Give the complete Smith form via the Kronecker canonical from. ⊚
5. Give a prime polynomial kernel spanning matrix via the Kronecker canonical form. ⊚
6. Give the complete behavior via the Kronecker canonical form. ⊚

Task 2: ⊚
Specify for each block in the Kronecker canonical form if it represents an autonomous system. If not give an input/output partition.

Task 3: ⊕
Define the matrices $A := \begin{bmatrix} 3 & 2 \\ 0 & 1 \end{bmatrix}$ and $B := \begin{bmatrix} 1 \\ 0 \end{bmatrix}$. Then, formulate the state-space system $\dot{x} = Ax + Bu$ in behavior form and give the Kronecker canonical form of the resulting first-order matrix polynomial.

Task 4: ⊖
For the system $\ddot{x} = Ax + Bu$ give a first order formulation which introduces less variables, than the (behavioral) canonical linearization (namely m less).

Task 5: ⊚
Assume that P is left prime and that U unimodular. Partition the rows of the product UP into the form $UP =: \begin{bmatrix} P_1 \\ P_2 \end{bmatrix}$. Show that P_1 and P_2 are also left prime.

Task 6: ⊖

1. Let $U \in \mathbb{C}[\lambda]^{p,p}$. Then U is unimodular if and only if the canonical linearization of U is unimodular.
2. Let $U \in \mathbb{C}[\lambda]^{p,q}$. Then U is (left/right) prime if and only if the canonical linearization of U is (left/right) prime.

Task 7: ⊚
Let $R \in \mathbb{C}[\lambda]^{p,q}$ have full row rank. Show that in this case a partition (P, Q) of R is input/output if and only if P is invertible.

Task 8: ⊕

Consider the example from the last series:

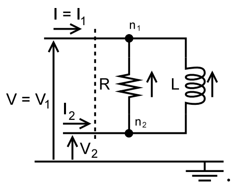

Figure 1: A simple RL circuit

With the definitions

$$P(\lambda) := \begin{bmatrix} 1 & 1 & 1 & 0 \\ -R & 0 & 0 & 1 \\ 0 & \lambda L & 0 & -1 \end{bmatrix} \qquad \text{and} \qquad z := \begin{bmatrix} I_R \\ I_L \\ I \\ V \end{bmatrix} \tag{1}$$

the system was given by $\mathcal{B}(P)$.

1. Give all input/output partitions of P.
2. Compute the Kronecker canonical from of P.

Task 9: ⊖

1. Let $A \in \mathbb{C}^{n,n}$. Show that $\lambda I - A \in \mathbb{C}[\lambda]_1^{n,n}$ is regular (over $\mathbb{C}(\lambda)$) by using the Jordan form and Lemma 1.9.
2. Use the canonical linearization to conclude that for every matrix polynomial $P(\lambda) = \sum_{i=0}^{K} \lambda^i P_i \in \mathbb{C}[\lambda]_K^{n,n}$ in which the highest coefficient P_K is invertible (over $\mathbb{C}$) we have that P is regular.
3. Conclude that for nonsquare polynomial matrices $P(\lambda) = \sum_{i=0}^{K} \lambda^i P_i \in \mathbb{C}[\lambda]^{p,q}$ in which the highest coefficient P_K has full (row or column) rank we have that also P has full (row or column) rank (over $\mathbb{C}(\lambda)$).
4. Give a counter example which shows that there exist polynomial matrices $P(\lambda) = \sum_{i=0}^{K} \lambda^i P_i \in \mathbb{C}[\lambda]^{p,q}$ which have full (row or column) rank (over $\mathbb{C}(\lambda)$) but in which the highest coefficient P_K does not have full (row or column) rank.

Task 10: ⊕

Let $A, E, M, D, K \in \mathbb{C}^{n,n}$, $B, B_1 \in \mathbb{C}^{n,m}$, and $D \in \mathbb{C}^{m,m}$. Let the dimensions of the involved functions be given by $x \in \mathcal{C}_\infty^n$ and $u \in \mathcal{C}_\infty^m$. Can you easily give an input/output partition for each of the following systems? Are the systems autonomuous?

1. $\dot{x}(t) = Ax(t) + Bu(t)$
2. $\ddot{x}(t) = Ax(t) + Bu(t)$
3. $E\dot{x}(t) = Ax(t)$ where $\lambda E - A \in \mathbb{C}[\lambda]^{n,n}$ is invertible/regular (over $\mathbb{C}(\lambda)$).
4. $E\dot{x}(t) = Ax(t)$ where $\lambda E - A \in \mathbb{C}[\lambda]^{n,n}$ is not invertible (over $\mathbb{C}(\lambda)$).
5. $M\ddot{x}(t) + D\dot{x}(t) + Kx(t) = Bu(t) + B_1\dot{u}(t)$ where M is invertible (over $\mathbb{R}$).
6. The following system with invertible M:

$$\begin{cases} M\ddot{x}(t) & = & Kx(t) & +Bu(t) \\ \dot{u}(t) & = & & Du(t) \end{cases}$$

Remark: Use the result from Task 9.