

Example 1.4.3 (Convergence of the mid-point rule)

Consider the initial value problem

$$\dot{y} = -5y \quad \text{with } y(0) = 1.$$

It has the exact solution $y(t) = e^{-5t}$. Use the explicit mid-point rule

$$u_{m+2} - u_m = 2hf_{m+1}$$

with (exact) start values $u_0 = 1$ and $u_1 = e^{-5h}$ for the numerical solution. For different stepsizes $h = 0.1, 0.05, 0.02, 0.01, 0.005$, we get the following results.

