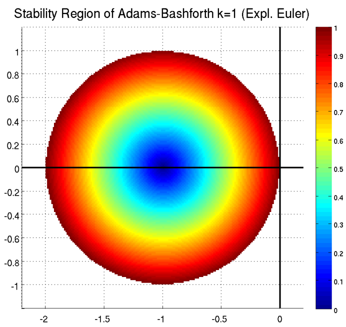
Example 1.4.4 (Stability Regions of Adams-Methods ((r,l)-Methods))

1. Adams-Bashforth k=1 (Explicit Euler method),



$$\alpha_0 = -1, \quad \alpha_1 = 1,$$
$$\beta_0 = 1, \quad \beta_1 = 0,$$

First char. pol.: $\rho(\xi) = \xi - 1$

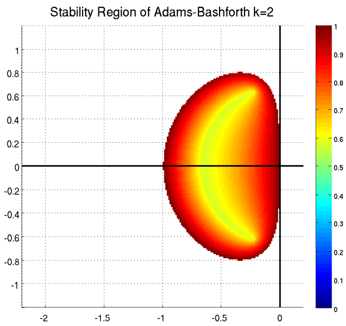
Second char. pol.: $\sigma(\xi) = 1$

Stability polynomial: $\Pi_z(\xi) = \rho(\xi) - z\sigma(\xi) = \xi - (1+z)$

Root: $\xi = (1+z)$ simple

Stability region: $S = \{z : |1+z| \leq 1\}$

2. Adams-Bashforth k=2,



$$\alpha_0 = 0, \quad \alpha_1 = -1, \quad \alpha_2 = 1,$$
$$\beta_0 = -\tfrac{1}{2}, \quad \beta_1 = \tfrac{3}{2}, \quad \beta_2 = 0,$$

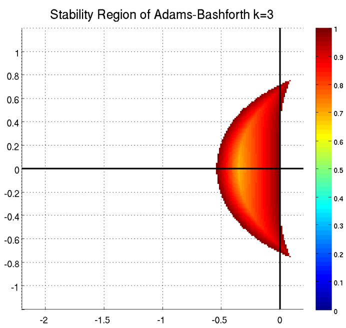
First char. pol.: $\rho(\xi) = \xi^2 - \xi$

Second char. pol.: $\sigma(\xi) = \frac{3}{2}\xi - \frac{1}{2}$

Stability polynomial: $\Pi_z(\xi) = \rho(\xi) - z\sigma(\xi) = \xi^2 + (-1 - \frac{3}{2}z)\xi + \frac{1}{2}z$

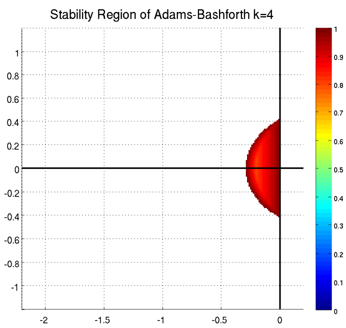
Root: $\xi = (\frac{1}{2} + \frac{3}{4}z) \pm \sqrt{\frac{1}{4} - \frac{1}{2}z + \frac{9}{16}z^2}$

3. Adams-Bashforth k=3,



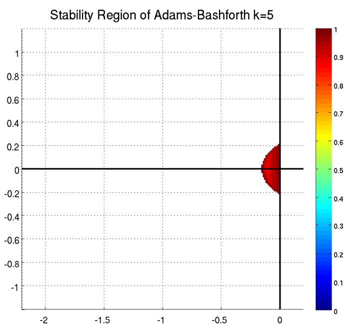
$$\alpha_0 = 0, \quad \alpha_1 = 0, \quad \alpha_2 = -1, \quad \alpha_3 = 1,$$
$$\beta_0 = \tfrac{5}{12}, \quad \beta_1 = -\tfrac{4}{3}, \quad \beta_2 = \tfrac{23}{12}, \quad \beta_3 = 0,$$

4. Adams-Bashforth k=4,



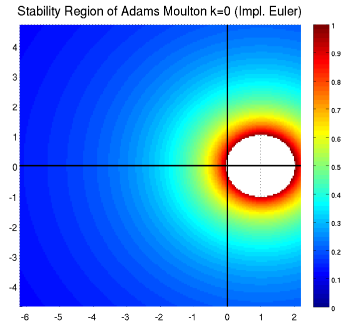
$$\alpha_0 = 0, \quad \alpha_1 = 0, \quad \alpha_2 = 0, \quad \alpha_3 = -1, \quad \alpha_4 = 1,$$
$$\beta_0 = -\tfrac{3}{8}, \quad \beta_1 = \tfrac{37}{24}, \quad \beta_2 = -\tfrac{59}{24}, \quad \beta_3 = \tfrac{55}{24}, \quad \beta_4 = 0,$$

5. Adams-Bashforth k=5,



$$\alpha_0 = 0, \quad \alpha_1 = 0, \quad \alpha_2 = 0, \quad \alpha_3 = 0, \quad \alpha_4 = -1, \quad \alpha_5 = 1,$$
$$\beta_0 = \tfrac{251}{720}, \quad \beta_1 = -\tfrac{637}{360}, \quad \beta_2 = \tfrac{109}{30}, \quad \beta_3 = -\tfrac{1387}{360}, \quad \beta_4 = \tfrac{1901}{720}, \quad \beta_5 = 0,$$

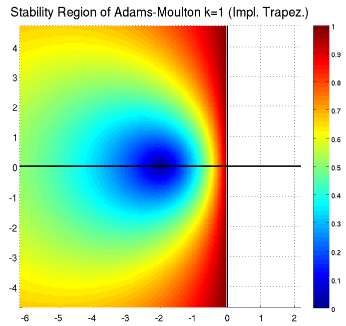
6. Adams-Moulton k=0 (Implicit Euler Method),



$$\alpha_{-1} = -1, \quad \alpha_0 = 1,$$
$$\beta_{-1} = 0, \quad \beta_0 = 1,$$

First char. pol.:	$\rho(\xi) = \xi - 1$
Second char. pol.:	$\sigma(\xi) = \xi$
Stability polynomial:	$\Pi_z(\xi) = \rho(\xi) - z\sigma(\xi)$
	$= (1-z)\xi - 1$
Root:	$\xi = 1/(1-z)$
Stability region:	$S = \{z : \|1-z\| \geq 1\}$

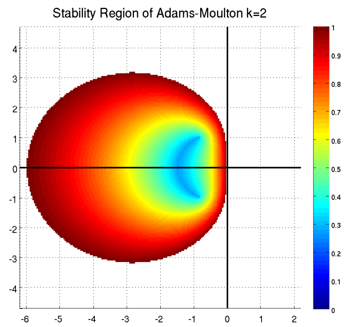
7. Adams-Moulton k=1 (Implicit Trapezoidal Rule),



$$\alpha_0 = -1, \quad \alpha_1 = 1,$$
$$\beta_0 = \tfrac{1}{2}, \quad \beta_1 = \tfrac{1}{2},$$

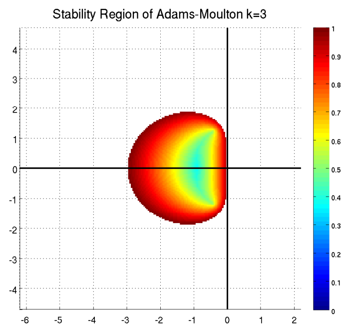
First char. pol.:	$\rho(\xi) = \xi - 1$
Second char. pol.:	$\sigma(\xi) = \frac{1}{2}\xi + \frac{1}{2}$
Stability polynomial:	$\Pi_z(\xi) = \rho(\xi) - z\sigma(\xi)$
	$= -(1 + z\frac{1}{2}) + (1 - z\frac{1}{2})\xi$
Root:	$\xi = (1 + \frac{1}{2}z)/(1 - \frac{1}{2}z)$
Stability region:	$S = \{z : Re(z) < 0\}$

8. Adams-Moulton k=2,



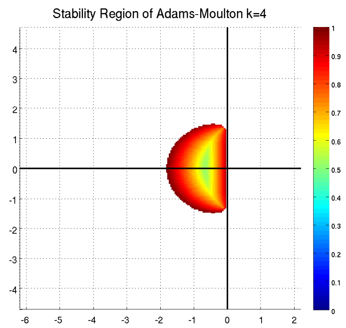
$$\alpha_0 = 0, \quad \alpha_1 = -1, \quad \alpha_2 = 1,$$
$$\beta_0 = -\tfrac{1}{12}, \quad \beta_1 = \tfrac{2}{3}, \quad \beta_2 = \tfrac{5}{12},$$

9. Adams-Moulton k=3,



$$\alpha_0 = 0, \quad \alpha_1 = 0, \quad \alpha_2 = -1, \quad \alpha_3 = 1,$$
$$\beta_0 = \tfrac{1}{24}, \quad \beta_1 = -\tfrac{5}{24}, \quad \beta_2 = \tfrac{19}{24}, \quad \beta_3 = \tfrac{3}{8},$$

10. Adams-Moulton k=4,



$$\alpha_0 = 0, \quad \alpha_1 = 0, \quad \alpha_2 = 0, \quad \alpha_3 = -1, \quad \alpha_4 = 1,$$
$$\beta_0 = -\tfrac{19}{720}, \quad \beta_1 = \tfrac{53}{360}, \quad \beta_2 = -\tfrac{11}{30}, \quad \beta_3 = \tfrac{323}{360}, \quad \beta_4 = \tfrac{251}{720},$$