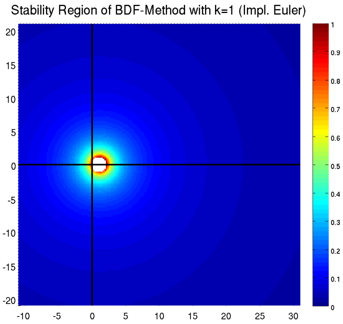
Example 1.4.6 (Stability Regions of BDF-Methods)

1. BDF k=1 (Implicit Euler Method),

$$hf(t_{m+1}, u_{m+1}) = u_{m+1} - u_m$$

$\alpha_0 = -1, \quad \alpha_1 = 1,$
$\beta_0 = 0, \quad \beta_1 = 1,$

First char. pol.:	$\rho(\xi) = \xi - 1$
Second char. pol.:	$\sigma(\xi) = \xi$
Stability polynomial:	$\Pi_z(\xi) = \rho(\xi) - z\sigma(\xi) = (1-z)\xi - 1$
Root:	$\xi = 1/(1-z)$
Stability region:	$S = \{z : \lvert 1-z \rvert \geq 1\}$

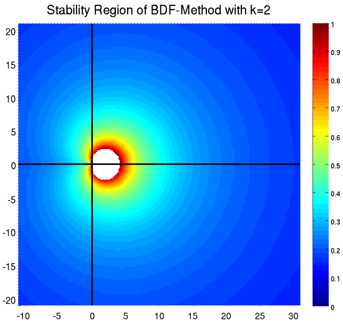


2. BDF k=2,

$$hf(t_{m+2}, u_{m+2}) = \tfrac{1}{2}(3u_{m+2} - 4u_{m+1} + u_m)$$

$\alpha_0 = \frac{1}{2}, \quad \alpha_1 = -\frac{4}{2}, \quad \alpha_2 = \frac{3}{2},$
$\beta_0 = 0, \quad \beta_1 = 0, \quad \beta_2 = 1,$

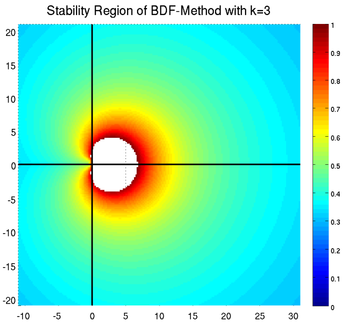
First char. pol.:	$\rho(\xi) = \frac{3}{2}\xi^2 - \frac{4}{2}\xi + \frac{1}{2}$
Second char. pol.:	$\sigma(\xi) = \xi^2$
Stability polynomial:	$\Pi_z(\xi) = \rho(\xi) - z\sigma(\xi) = (\frac{3}{2} - z)\xi^2 - \frac{4}{2}\xi + \frac{1}{2}$



3. BDF k=3,

$$hf(t_{m+3}, u_{m+3}) = \tfrac{1}{6}(11u_{m+3} - 18u_{m+2} + 9u_{m+1} - 2u_m)$$

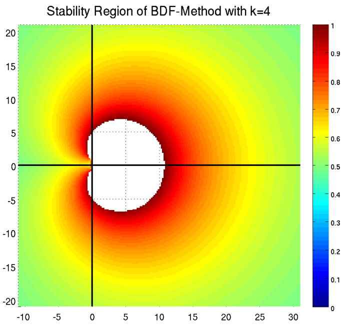
$\alpha_0 = -\frac{2}{6}, \quad \alpha_1 = \frac{9}{6}, \quad \alpha_2 = -\frac{18}{6}, \quad \alpha_3 = \frac{11}{6},$
$\beta_0 = 0, \quad \beta_1 = 0, \quad \beta_2 = 0, \quad \beta_3 = 1,$



4. BDF k=4,

$$hf(t_{m+4}, u_{m+4}) = \tfrac{1}{12}(25u_{m+4} - 48u_{m+3} + 36u_{m+2} - 16u_{m+1} + 3u_m)$$

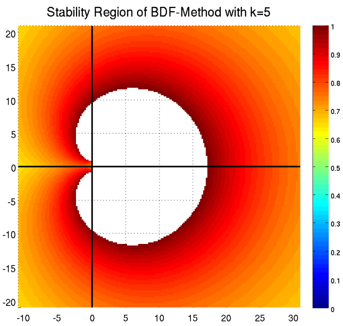
$\alpha_0 = \frac{3}{12}, \quad \alpha_1 = -\frac{16}{12}, \quad \alpha_2 = \frac{36}{12}, \quad \alpha_3 = -\frac{48}{12}, \quad \alpha_4 = \frac{25}{12},$
$\beta_0 = 0, \quad \beta_1 = 0, \quad \beta_2 = 0, \quad \beta_3 = 0, \quad \beta_4 = 1,$



5. BDF k=5,

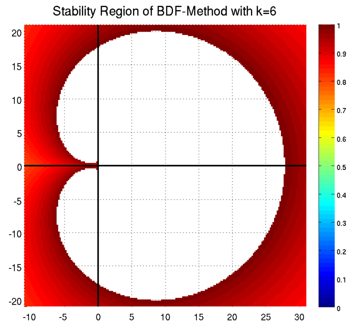
$$hf(t_{m+5}, u_{m+5}) = \tfrac{1}{60}(137u_{m+5} - 300u_{m+4} + 300u_{m+3} - 200u_{m+2} + 75u_{m+1} - 12u_m)$$

$\alpha_0 = -\frac{12}{60}, \quad \alpha_1 = \frac{75}{60}, \quad \alpha_2 = -\frac{200}{60}, \quad \alpha_3 = \frac{300}{60}, \quad \alpha_4 = -\frac{300}{60}, \quad \alpha_5 = \frac{137}{60},$
$\beta_0 = 0, \quad \beta_1 = 0, \quad \beta_2 = 0, \quad \beta_3 = 0, \quad \beta_4 = 0, \quad \beta_5 = 1,$



6. BDF k=6,

$$hf(t_{m+6}, u_{m+6}) = \frac{1}{60}(147u_{m+6} - 360u_{m+5} + 450u_{m+4} - 400u_{m+3} + 225u_{m+2} - 72u_{m+1} + 10u_m)$$



$\alpha_0 = \frac{10}{60}$, $\alpha_1 = -\frac{72}{60}$, $\alpha_2 = \frac{225}{60}$, $\alpha_3 = -\frac{400}{60}$, $\alpha_4 = \frac{450}{60}$, $\alpha_5 = -\frac{360}{60}$, $\alpha_6 = \frac{147}{60}$,
$\beta_0 = 0$, $\beta_1 = 0$, $\beta_2 = 0$, $\beta_3 = 0$, $\beta_4 = 0$, $\beta_5 = 0$, $\beta_6 = 1$,

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