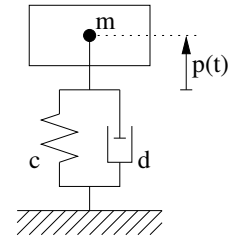**Example 1.2.6 (Linear spring-damper system)**
Consider a linear spring-damper system with its model equation

$$\dot{y} = \begin{bmatrix} 0 & 1 \\ -c/m & -d/m \end{bmatrix} y$$

on $t \in [t_0, t_f] = [0, 10]$. We choose the mass $m = 1$, the damping $d = 1$ and the stiffness c varying in $[0, 100]$. The initial condition is

$$y(t_0) = y_0 = \begin{bmatrix} p_0 \\ v_0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$$

The numerical results by use of the explicit Euler method and the implicit Euler method with an equidistant stepsize $h = 1/20$ as well as the stability region of the explicit Euler method and the h-scaled eigenvalues are illustrated in the following figures for different c.

c=20

Exact Solution and Numerical Solution with the Explicit Euler Method

Exact Solution and Numerical Solution with the Implicit Euler Method

Position of $h\lambda_i$ and Stability Region
Position of $h\lambda_i$ and Stability Region
Position of $h\lambda_i$ and Stability Region

$\lambda_1 = -0.5 + 4.4441i$ $h\lambda_1 = -0.025 + 0.2222i$ Distance of $h\lambda_1$ to the center of circle $=1$
$\lambda_2 = -0.5 - 4.4441i$ $h\lambda_2 = -0.025 - 0.2222i$ Distance of $h\lambda_2$ to the center of circle $=1$

c=21

Exact Solution and Numerical Solution with the Explicit Euler Method

Exact Solution and Numerical Solution with the Implicit Euler Method

Position of $h\lambda_i$ and Stability Region
Position of $h\lambda_i$ and Stability Region
Position of $h\lambda_i$ and Stability Region

$\lambda_1 = -0.5 + 4.5552i$ $h\lambda_1 = -0.025 + 0.22776i$ Distance of $h\lambda_1$ to the center of circle $=1.0012$
$\lambda_2 = -0.5 - 4.5552i$ $h\lambda_2 = -0.025 - 0.22776i$ Distance of $h\lambda_2$ to the center of circle $=1.0012$

c=25

Exact Solution and Numerical Solution with the Explicit Euler Method

Exact Solution and Numerical Solution with the Implicit Euler Method

Position of $h\lambda_i$ and Stability Region
Position of $h\lambda_i$ and Stability Region
Position of $h\lambda_i$ and Stability Region

$\lambda_1 = -0.5 + 4.9749i$ $h\lambda_1 = -0.025 + 0.24875i$ Distance of $h\lambda_1$ to the center of circle $=1.0062$
$\lambda_2 = -0.5 - 4.9749i$ $h\lambda_2 = -0.025 - 0.24875i$ Distance of $h\lambda_2$ to the center of circle $=1.0062$

$\triangleleft$