

Example 1.4.2 (Instabil behavior)

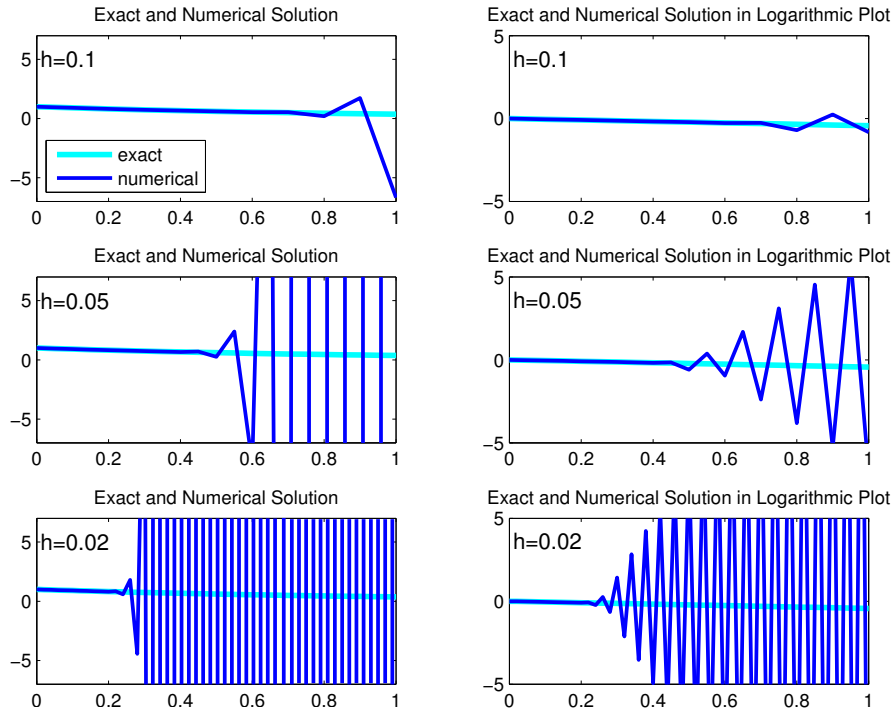
Consider the initial value problem

$$\dot{y} = -y \quad \text{with } y(0) = 1.$$

It has the exact solution $y(t) = e^{-t}$. Use the explicit two-step method

$$u_{m+2} + 4u_{m+1} - 5u_m = h(4f_{m+1} + 2f_m)$$

with (exact) start values $u_0 = 1$ and $u_1 = e^{-h}$ for the numerical solution. For different stepsizes $h = 0.1, 0.05, 0.02$ we get the following results.



| t | ex.Sol. | h=0.1 | h=0.05 | h=0.02 |
|------------|------------|-------------|-------------|-------------|
| 0 | 1.0000e+00 | 1.0000e+00 | 1.0000e+00 | 1.0000e+00 |
| 1.0000e-01 | 9.0484e-01 | 9.0484e-01 | 9.0484e-01 | 9.0484e-01 |
| 2.0000e-01 | 8.1873e-01 | 8.1872e-01 | 8.1871e-01 | 8.1081e-01 |
| 3.0000e-01 | 7.4082e-01 | 7.4087e-01 | 7.4032e-01 | 2.7008e+01 |
| 4.0000e-01 | 6.7032e-01 | 6.7000e-01 | 6.5699e-01 | -8.7146e+04 |
| 5.0000e-01 | 6.0653e-01 | 6.0820e-01 | 2.5293e-01 | 2.8913e+08 |
| 6.0000e-01 | 5.4881e-01 | 5.3991e-01 | -8.8339e+00 | -9.5923e+11 |
| 7.0000e-01 | 4.9659e-01 | 5.4377e-01 | -2.4848e+02 | 3.1825e+15 |
| 8.0000e-01 | 4.4933e-01 | 1.9897e-01 | -6.6061e+03 | -1.0558e+19 |
| 9.0000e-01 | 4.0657e-01 | 1.7346e+00 | -1.7530e+05 | 3.5030e+22 |
| 1.0000e+00 | 3.6788e-01 | -6.6773e+00 | -4.6517e+06 | -1.1622e+26 |