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Summer Term 2014

Numerical Analysis II

Homework Sheet 1

No Exercises and No Theoretical Homework

Tere will be a Lecture instead of the Tutorial on April 23

Programming Homework

Due: April 28 (first chance) or May 5 (second chance)

Attention: You can only submit your program on May 5 if you presented a programming approach on April 28!

Write a program that solves an ordinary differential equation $\dot{y}(t) = f(t, y(t))$, $y(t_0) = y_0$ on an interval $[t_0, t_0 + a]$ using an explicit one-step method. Your program should be called with the line

$$[\mathtt{h}, \mathtt{t}, \mathtt{u}] = \mathtt{expode}(\mathtt{i}, \mathtt{fun}, \mathtt{t0}, \mathtt{y0}, \mathtt{N}, \mathtt{a}),$$

where the paramenter `i` indicates the method that is used (`i`=1 − Forward Euler, `i`=2 − Heun's method, `i`=3 − Collatz method).
Moreover, `fun` should be a MATLAB function handle corresponding to the right hand side $f(t, y)$ of the differential equation. For each right hand side, you therefore need a file `funct?.m` which is called via

$$[\mathtt{f}] = \mathtt{funct?}(\mathtt{t}, \mathtt{y}),$$

where $? = 1, 2$. It should be possible here for `y` and `f` to be vectors of $\mathbb{R}^m$.
The parameter $\mathtt{t0} = t_0$ is the lower interval bound, $\mathtt{y0} = y_0 \in \mathbb{R}^m$ is the initial value, $\mathtt{N} = N$ is the number of (equidistant) grid points, and $\mathtt{a} = a$ is the interval length. The routine should return the (constant) step size $\mathtt{h} = h = a/N$, the vector of grid points $\mathtt{t} = [t_0, t_1, \ldots t_N]$, and the corresponding approximated solution $\mathtt{u} = [u_0, u_1, \ldots u_N]$.
To test your program, use the initial value problems

1. $\dot{y}(t) = -y\cos(t), \quad y(0) = 1$
   The solution is $y(t) = e^{-\sin(t)}$,

2. $$\begin{aligned} \dot{y}_1 &= y_2 & &\text{with} \quad y_1(0) = \sin(1), \\ \dot{y}_2 &= -y_1 e^{2t} + y_2 & &\text{with} \quad y_2(0) = \cos(1) \end{aligned}$$
   The solution is $y_1 = \sin(e^t)$, $y_2 = \cos(e^t)e^t$,

and approximate the solution for $a = 2$ using $N = 5, 10, 100, 1000$ steps.
Make twelve plots: four rows for the different values of N and three columns for the test functions. Every plot should contain the errors $\mathtt{e}{=}\mathbf{e} = [e_0, e_1, \ldots, e_N]$, with $e_i = ||y(t_i) - \mathtt{u(i)}||_\infty$, that are made in the approximated solutions for all three explicit one-step methods. Use a logarithmic scale on the y-axis. Give an interpretation of the results.