

Numerical Analysis II

Homework Sheet 2

Exercises

Tutorial on April 28

1. Problem

Derive Heun's method by numerical integration of

$$y'(t) = f(t, y(t)), \quad y(0) = y_0,$$

on a uniform grid of size h using the trapezoidal quadrature rule. Moreover, give a geometrical interpretation of Heun's method.

2. Problem

Show that Heun's method is consistent of order 2.

3. Problem

Which conditions on the entries of the general Butcher's tableau

$$\begin{array}{c|cc} 0 & & \\ \hline c_2 & a_{21} & \\ \hline & b_1 & b_2 \end{array} \quad (1)$$

should hold, such that the corresponding explicit Runge-Kutta method is consistent of order two?

Theoretical Homework

Due: May 6, during the lecture

1. Problem

Show that Collatz' method is consistent of order 2. Is this method also convergent?

(5 Points)

2. Problem

Take the Butcher's tableau (1). In the tutorial, we have derived conditions for which it holds

(5 Points)

$$\|\tau(t, h, f)\| \leq Ch^3$$

for the corresponding Runge-Kutta method. Here, τ is the local discretization error and C is an h -independent constant. Determine those parameters a_{21} , b_1 , b_2 , and c_2 , for which constant C is as small as possible.

Total Points: 10