Technische Universität Berlin **Summer Term 2014**
Fakultät II - Mathematik und Naturwissenschaften
Institut für Mathematik
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Numerical Analysis II
Homework Sheet 4

Exercises
Tutorial on May 12

1. **Problem**
Show that if the solution of a linear system of ODEs $\dot{x} = Ax$ has a "damped" behavior, then the solutions of the scalar ODEs $\dot{y} = \lambda_i y$ also have a "damped" behavior for all eigenvalues λ_i of A.

2. **Problem**
Show that the solution $y(t)$ of the linear ordinary differential equation $\dot{y}(t) = Ay(t)$, $y(t_0) = y_0$, is stable if and only if $\mathrm{Re}(\lambda) \leq 0$ for all eigenvalues of A and $\mathrm{Re}(\lambda) < 0$ for eigenvalues of A where the geometric and algebraic multiplicities do not coincide.

3. **Problem**
Compute the stability function of Heun's method and of the implicit trapezoidal rule. Compare the stability regions of these two methods.

Theoretical Homework
Due: May 20, during the lecture

1. **Problem** (5 Points)
Compute the stability function of Simpson's rule (order 3), the Runge method (order 3), and the implicit Radau IIA method (order 3). The Butcher tableaus can be found on the back side. Draw a sketch of the three stability regions − printouts of MATLAB plots are fine if you submit your code as well.

2. **Problem** (10 Points)
Prove the following Lemma:

Lemma. *The stability function of a Runge-Kutta method is given by*

$$R(z) = \frac{\det(I - zA + z\mathbb{1}b^T)}{\det(I - zA)}.$$

From this characterization, what can you infer about the stability regions of explicit Runge-Kutta methods and their suitability for stiff problems?
Hint: Cramer's rule.

3. **Problem** (5 Points)
Show that the backward Euler method corresponds to the Butcher tableau

$$\begin{array}{c|c} 1 & 1 \\ \hline & 1 \end{array}.$$

What tableau does the implicit trapezoidal rule correspond to? Prove your answer.

Total Points: 20

Last updated on May 8, 2014.

Simpson's rule (3):

0			
1/2	1/2		
1	−1	2	
	1/6	4/6	1/6

Runge method (3):

0				
1/2	1/2			
1	0	1		
1	0	0	1	
	1/6	2/3	0	1/6

Radau IIA (3):

1/3	5/12	−1/12
1	3/4	1/4
	3/4	1/4