

# Numerical Analysis II

## Homework Sheet 6

### Exercises

#### Tutorial on May 22

#### 1. Problem

Consider the extrapolation of a basic method that is consistent of order  $p$  with  $s = 2$  stages using the series  $(n_i)_{i=1}^2 = (1, 2)$ . What formula do you receive?

#### 2. Problem

Compute the adjoint methods of Heun's method, Collatz' method, the implicit trapezoidal rule, and the explicit midpoint rule (leap frog method)

$$u_{m+1} = u_{m-1} + 2hf(t_m, u_m).$$

Which of these methods are selfadjoint?

#### 3. Problem

Compute the adjoint of the Radau IIA method, which is given by the Butcher tableau

$$\begin{array}{c|cc} 1/3 & 5/12 & -1/12 \\ 1 & 3/4 & 1/4 \\ \hline & 3/4 & 1/4 \end{array}.$$

### Theoretical Homework

#### Due: June 3, during the lecture

#### 1. Problem

(10 Points)

Show that the adjoint of an  $s$ -stage Runge-Kutta method is again an  $s$ -stage Runge-Kutta method with the Butcher tableau

$$\begin{array}{c|cccc} (1-c_1) & (b_1-a_{11}) & (b_2-a_{12}) & \dots & (b_s-a_{1s}) \\ \vdots & \vdots & \vdots & & \vdots \\ (1-c_s) & (b_1-a_{s1}) & (b_2-a_{s2}) & \dots & (b_s-a_{ss}) \\ \hline & b_1 & b_2 & \dots & b_s \end{array}.$$

#### 2. Problem

(10 Points)

Show that if we have for the coefficients of a consistent  $s$ -stage Runge-Kutta method that

$$a_{s+1-i, s+1-j} + a_{ij} = b_{s+1-j} = b_j, \quad i, j = 1, 2, \dots, s,$$

then it is selfadjoint (you may assume that the node conditions are satisfied).

Total Points: 20