Technische Universität Berlin **Summer Term 2014**
Fakultät II - Mathematik und Naturwissenschaften
Institut für Mathematik
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Numerical Analysis II

Homework Sheet 8

Exercises

Tutorial on June 16

1. Problem
Show that the set of solutions to the homogeneous difference equation $u_{m+k} + \sum_{i=0}^{k-1} \gamma_i u_{m+i} = 0$ is a vector space.

2. Problem
Prove the following Theorem.

Theorem (Discrete Duhamel's Principle). *The solution to the* inhomogeneous *difference equation*

$$\begin{cases} u_{m+2} + \gamma_1 u_{m+1} + \gamma_0 u_m = f_{m+2}, & m \geq 0 \\ u_0 = 0, \quad u_1 = f_1 \end{cases}$$

is given by $u_{m+2} = \sum_{j=0}^{m+1} q_{m+1}^{(j)}$, *where* $q_m^{(j)}$ *is the solution of the* homogeneous *difference equation*

$$\begin{cases} q_{m+2}^{(j)} + \gamma_1 q_{m+1}^{(j)} + \gamma_0 q_m^{(j)} = 0, & m \geq j \\ q_j^{(j)} = f_{j+1}, \quad q_{j+1}^{(j)} = -\gamma_1 f_{j+1} \end{cases}$$

for each $j = 0, 1, 2, \ldots$

3. Problem
Solve the difference equation

$$\begin{cases} u_{m+2} - 3u_{m+1} + 2u_m = m + 2, & m \geq 0, \\ u_0 = 0, \quad u_1 = 1. \end{cases}$$

Theoretical Homework

Due: June 24, during the lecture

1. Problem (7 Points)
Solve the following linear difference equations

(a) $u_{m+2} - 2u_{m+1} - 3u_m = 0, \qquad u_0 = 0, \quad u_1 = 1,$
(b) $u_{m+2} - 4u_{m+1} + 3u_m = m + 2, \quad u_0 = 0, \quad u_1 = 1.$

2. Problem (7 Points)
Consider the inhomogeneous linear difference equation

$$u_{m+k} + \gamma_{k-1} u_{m+k-1} + \cdots + \gamma_0 u_m = f_m,$$

where $u_0, \ldots, u_{k-1}, f_m \in \mathbb{R}$ are given for all $m \in \mathbb{N}$.

(a) Show that the above difference equation can be transformed to a system of linear difference equations of order one of the form
$$U_{m+1} = AU_m + F_m.$$
How are U_m, A, and F_m defined?
(b) Determine a general formula for the solution U_m depending on U_0 using (a).
(c) Solve the difference equation $u_{m+2} - 4u_{m+1} - 5u_m = 1$, $u_0 = 0$, $u_1 = 1$ using (b).

3. Problem (6 Points)

Show that the characteristic polynomial $p(\lambda) = \det(\lambda I - M)$ of the $k \times k$ matrix

$$M = \begin{bmatrix} 0 & 1 & & \\ & \ddots & \ddots & \\ & & 0 & 1 \\ -\alpha_0 & \cdots & -\alpha_{k-2} & -\alpha_{k-1} \end{bmatrix}$$

is given by $p(\lambda) = \lambda^k + \alpha_{k-1}\lambda^{k-1} + \ldots + \alpha_1\lambda + \alpha_0$.

Total Points: 20