Technische Universität Berlin
Fakultät II - Mathematik und Naturwissenschaften
Institut für Mathematik
Andreas Steinbrecher, Jeroen Stolwijk

Summer Term 2014

Numerical Analysis II

Homework Sheet 10

Exercises

Tutorial on June 30

1. **Problem**
   Determine the stability regions of the $P(EC)^M E$ methods using forward Euler as the predictor and the implicit trapezoidal rule as the corrector. What happens for $M \to \infty$?

2. **Problem**
   Compute the coefficients of the k-step BDF method for $k = 1, 2, 3$.

Theoretical Homework

Due: July 10, during the lecture

1. **Problem** (8 Points)
   Consider the $P(EC)^M E$ method that is achieved by using

$$u_{m+k} + \sum_{l=0}^{k-1} \alpha_l^P u_{m+l} = h \sum_{l=0}^{k-1} \beta_l^P f_{m+l}$$

as the predictor and

$$u_{m+k} + \sum_{l=0}^{k-1} \alpha_l^C u_{m+l} = h \sum_{l=0}^{k} \beta_l^C f_{m+l}$$

as the corrector. Let further be

$$\sum_{l=0}^{k} \gamma_l^{(M)} u_{m+l} = 0$$

the difference equation that is obtained by applying the above predictor-corrector method to the test equation $f(t, u) = \lambda u$. Derive a recursive formula for the coefficients $\gamma_l^{(M)}$ for $l = 0, \ldots, k$, $M \in \mathbb{N}$ and prove it (by induction).

2. **Problem** (12 Points)
   Sketch the stability regions of the following multi-step methods:

   (a) $u_{m+3} = u_{m+2} + \frac{h}{12}(23f_{m+2} - 16f_{m+1} + 5f_m)$ (Adams-Bashforth, $k = 3$)

   (b) $u_{m+3} = u_{m+2} + \frac{h}{24}(9f_{m+3} + 19f_{m+2} - 5f_{m+1} + f_m)$ (Adams-Moulton, $k = 3$)

   (c) $P(EC)^M E$: Adams-Bashforth/Adams-Moulton for $k = 3$ and $M = 2$, use problem 1.

   (d) $\frac{1}{6}(11u_{m+3} - 18u_{m+2} + 9u_{m+1} - 2u_m) = hf_{m+3}$ (BDF, $k = 3$)

   (e) $\frac{1}{60}\big(147u_{m+6} - 360u_{m+5} + 450u_{m+4} + 400u_{m+3} + 225u_{m+2} - 72u_{m+1} + 10u_m\big) = hf_{m+6}$ (BDF, $k = 6$)

   Again, printouts of MATLAB plots are fine if you submit your code as well.

   Hint: Use the MATLAB function `roots`.

Total Points: 20

Last updated on June 27, 2014.