ADM III: Fortgeschrittene Methoden der Ganzzahligen Linearen Programmierung
Exercise sheet 1

deadline: 26.04.2007, 8:30

All exercises deal with the Asymmetric Traveling Salesman Problem (ATSP). The ATSP is supposed to be defined on the complete directed graph on n nodes $D_n = (V, A_n)$ with $V = \{1, \ldots, n\}$ and $A_n = \{(i,j) \mid i,j \in V, i \neq j\}$.

Exercise 1 — 4 points

The following equalities are valid for the ATSP-polytope P_T^n

$$x(\delta^-(v)) = 1, \quad \forall v \in V$$
$$x(\delta^+(v)) = 1. \quad \forall v \in V$$

Show that the rank of this system is $2n-1$, i. e., one equality is redundant.

Note: The notation used means the following:

$$x(A) := \sum_{a \in A} x_a \qquad \text{for } A \subseteq A_n$$
$$\delta^-(i) := \{(j,i) \in A_n \mid j \in V\} \qquad \text{for } i \in V$$
$$\delta^+(i) := \{(i,j) \in A_n \mid j \in V\} \qquad \text{for } i \in V$$

Exercise 2 — 4 points

Show that $x_{ij} \leq 1$ is not a facet of P_T^n.

Exercise 3 — 2+2 points

Determine and prove the dimension of P_T^3 and P_T^4.

Exercise 4 — bonus points

Generalize the results of the last exercise and prove a general result for the dimension of P_T^n.