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Exercise sheet 2

Deadline: Thursday, May 3rd, 2007, 08:30 h in MA-313

Exercise 1: 4 points

a) Show that the integer linear program (CYC*) is **not** a correct ATSP formulation:

$$\begin{aligned} & \min c^T x & & \text{(CYC*)} \\ s.t.\quad & \mathbf{1}^T x = n & & \\ & x(C) \leq |C| - 1 & & \forall \text{ cycles } C \subseteq A,\ |C| < n \\ & 0 \leq x_a \leq 1 & & \forall\, a \in A \\ & \mathbf{x} \in \mathbb{Z}^A & & \end{aligned}$$

b) Show that the integer linear program (CYC) is a correct ATSP formulation:

$$\begin{aligned} & \min c^T x & & \text{(CYC)} \\ s.t.\quad & x(\delta^+(v)) = 1 & & \forall\, v \in V \\ & x(\delta^-(v)) = 1 & & \forall\, v \in V \\ & x(C) \leq |C| - 1 & & \forall \text{ cycles } C \subseteq A,\ |C| < n \\ & 0 \leq x_a \leq 1 & & \forall\, a \in A \\ & \mathbf{x} \in \mathbb{Z}^A & & \end{aligned}$$

Exercise 2: 4 points

Show that the *cut formulation* (CUT) and and the *subtour formulation* (SUB) for ATSP are equally strong, (i.e., that their linear programming relaxations are equivalent.)

$$\begin{aligned} & \min c^T x & & \text{(CUT)} \\ s.t.\quad & x(\delta^+(v)) = 1 & & \forall\, v \in V \\ & x(\delta^-(v)) = 1 & & \forall\, v \in V \\ & x(\delta^+(W)) \geq 1 & & \forall\, \emptyset \neq W \subsetneq V \\ & 0 \leq x_a \leq 1 & & \forall\, a \in A \\ & \mathbf{x} \in \mathbb{Z}^A & & \end{aligned}$$

$$\begin{aligned}
&\min c^T x && && \text{(SUB)}\\
s.t.\quad & x(\delta^+(v)) = 1 && \forall\, v \in V\\
& x(\delta^-(v)) = 1 && \forall\, v \in V\\
& x(A(W)) \leq |W| - 1 && \forall\, \emptyset \neq W \subsetneq V && \text{(*)}\\
& 0 \leq x_a \leq 1 && \forall\, a \in A\\
& \mathbf{x} \in \mathbb{Z}^A
\end{aligned}$$

Hint: Show that every solution x^* of the linear relaxation of (CUT) is also a solution of the linear relaxation of (SUB) and vice versa.

Exercise 3: **4 points**

Show that the linear relaxation of the cut formulation (CUT) (or, equivalently, of the subtour formulation (SUB)) is stonger than the linear relaxation of the cycle formulation (CYC).

Hint:

- Show that every solution x^* of the linear relaxation of (CUT) is also a solution of the linear relaxation of (CYC).
- Construct an ATSP instance and a solution x^* of the linear relaxation of (CYC) such that x^* violates some of the subtour constraints (*).

Exercise 4: **4 points**

Show that the linear relaxation of the cut formulation (CUT) (or, equivalently, of the subtour formulation (SUB)) is stronger than the linear relaxation of the *Miller-Tucker-Zemlin formulation* (MTZ).

$$\begin{aligned}
&\min c^T x && && \text{(MTZ)}\\
s.t.\quad & x(\delta^+(v)) = 1 && \forall\, v \in V\\
& x(\delta^-(v)) = 1 && \forall\, v \in V\\
u_v - u_w + (n-1)\, & x_{(v,w)} \leq n - 2 && \forall\, (v,w) \in A, w \neq 1\\
& 0 \leq x_a \leq 1 && \forall\, a \in A\\
& 1 \leq u_v \leq n-1 && \forall\, v \in V \setminus \{1\}\\
& u_1 = 0\\
& \mathbf{x} \in \mathbb{Z}^A\\
& \mathbf{u} \in \mathbb{Z}^V
\end{aligned}$$