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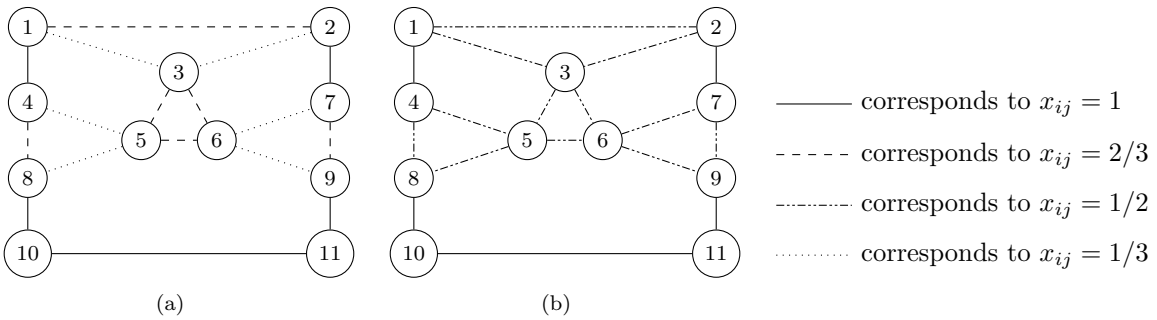
Exercise sheet 3

Deadline: Thursday, May 10th, 2007, 08:30 h in MA-313

Exercise 1: 4 points



a) Find a comb inequality which is violated by the point $x \in \mathbb{R}^{55}$ shown in (a).

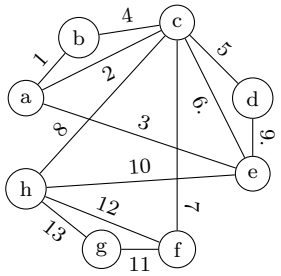
b) Find a clique tree inequality which is violated by the point $x \in \mathbb{R}^{55}$ shown in (b).

Exercise 2: 4 points

Let $(E, \mathcal{I})$ be an independence system with rank function r and $F \subseteq E$. Prove the following fact: If the inequality $x(F) \leq r(F)$ defines a facet of $P_{\mathcal{I}}$, then F is closed and inseparable.

Exercise 3: 4 points

The following graph G on 8 nodes with 13 edges has no hamiltonian circuit. Prove this fact using polyhedral theory.



Hint: Define

$$\mathrm{TSP}(G) := \mathrm{conv}\{\chi^T \in \mathbb{R}^{13} \mid T \text{ hamiltonian circuit}\}$$

and find a system of inequalities valid for $\mathrm{TSP}(G)$. Then show that this system has no solution using the Farkas Lemma.

Exercise 4: **4 points**

Let C be a cycle of length $3 \leq k \leq n-1$ in D_n and consider the corresponding cycle inequality $x(C) \leq |C| - 1$. Let (i, j) be a diagonal of C. Determine the coefficient a_{ij} for lifting (i, j) into the cycle inequality.