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Exercise sheet 5

Deadline: Thursday, May 31st, 2007, 08:30 h in MA-313

Exercise 1: **4 points**

Given an undirected graph $G = (V, E)$, the corresponding stable set polytope $\text{STAB}(G) \subseteq \mathbb{R}^V$ is defined as

$$\begin{aligned} \text{STAB}(G) :=& \text{conv}\{\, \chi^S \mid S \subseteq V \text{ is a stable set} \,\} \\ =& \text{conv}\{\, x \in \{0,1\}^V \mid x_i + x_j \leq 1 \quad \forall ij \in E \,\} \ . \end{aligned}$$

For any odd cycle C in G, the so-called *odd hole inequality*

$$x(V(C)) \leq \frac{|C| - 1}{2} \tag{1}$$

is valid for $\text{STAB}(G)$. Prove the following two facts:

a) If G itself is an odd cycle (without diagonals), then the corresponding odd hole inequality (1) defines a facet of $\text{STAB}(G)$.

b) If G is an odd cycle with an additional diagonal edge, then the corresponding odd hole inequality (1) (for the cycle C with $V(C) = V(G)$) is not facet-defining for $\text{STAB}(G)$.

Exercise 2: **4 points**

Let $G = (V, E)$ be an undirected graph and $\text{STAB}(G) \subseteq \mathbb{R}^V$ be defined as in Exercise 1. For any clique $C \subseteq V$ in G, the so-called *clique inequality*

$$x(C) \leq 1 \tag{2}$$

is valid for $\text{STAB}(G)$. Prove the following two facts:

a) Inequality (2) defines a facet of $\text{STAB}(G)$ if and only if C is an inclusion-wise maximal clique.

b) If $|C| \geq 4$, then inequality (2) is not contained in the first Chvátal-Gomory closure $e^1(P(G))$ of the fractional stable set polytope

$$\begin{aligned} P(G) := \{\, x \in \mathbb{R}^V \mid\ & x_i + x_j \leq 1 \quad \forall ij \in E, \\ & x_i \geq 0 \quad \forall i \in V \,\}. \end{aligned}$$

Exercise 3: **4 points**

An *interval matrix*, also called *consecutive ones* matrix, is a 0/1-matrix such that in each row the 1-entries are consecutive. Prove that interval matrices are totally unimodular.

Exercise 4: **4 points**

Show that the matrix

$$A := \begin{pmatrix} 1 & 1 & 1 \\ -1 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

is not totally unimodular, but that nevertheless

$$P^{=}(A, b) := \{\, x \mid Ax = b \,\}$$

is integral for all integral vectors b.