Technische Universität Berlin
Institut für Mathematik

ADM III – Advanced Methods for Integer Linear Programming
Summer Term 2007

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Exercise sheet 7

Deadline: Thursday, June 14th, 2007, 08:30 h in MA-313

Exercise 1: **4 points**

Let $(X, \leq)$ be a partially ordered set. $a, b \in X$ are called *comparable* if $a \leq b$ or $b \leq a$ and *incomparable* otherwise. A *chain* (*antichain*) is a subset $S \subseteq X$, where all elements of S are pairwise comparable (incomparable). Prove the following theorem.

Theorem 1 (Dilworth's Theorem) *In any finite partially ordered set $(X, \leq)$ the maximum size of an antichain is equal to the minimum number of chains needed to cover X.*

Hint: Use total unimodularity of some suitable network matrices.

Exercise 2: **4 points**

A graph $G = (V, E)$ is an *interval graph*, if there is an *interval representation* $I = \{[a_i, b_i]\}_{i \in V}$ such that

$$\{i, j\} \in E \iff [a_i, b_i] \cap [a_j, b_j] \neq \emptyset,$$

where $[a_i, b_i]$ are closed intervals.

Prove that interval graphs are perfect using Dilworth's Theorem.

Exercise 3: **4 points**

The line graph of a graph $G = (V, E)$ is the graph with node set E and two nodes are adjacent if they have a common end node in G.

Prove that the line graph of a bipartite graph is perfect using König's Edge Coloring Theorem.

Theorem 2 (König's Edge Coloring Theorem) *If G is a bipartite graph then the maximum degree $\Delta(G)$ is equal to the minimum number of colors needed to color the edges such that no two adjacent edges have the same color.*

Exercise 4: **4 points**

Describe an algorithm that for an arbitrary rational symmetric matrix A decides whether A is positive semidefinite with running time polynomial in the encoding length of A.

Hint: A matrix A is *positive semidefinite* if and only if for all vectors x we have $x^T Ax > 0$. There may be other characterizations more suited to checking positive semidefiniteness.