Technische Universität Berlin
Institut für Mathematik

ADM III – Advanced Methods for Integer Linear Programming
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Exercise sheet 8

Deadline: Thursday, June 21th, 2007, 08:30 h in MA-313

Exercise 1: **4 points**

For a set $P \subseteq \mathbb{R}^n$ of antiblocking type, the set

$$A(P) = \{z \in \mathbb{R}^n \mid z^T x \leq 1 \; \forall x \in P\}$$

is called the *antiblocker of* P. Show that the antiblocker of the stable set polytope STAB(G) for a graph G is the polytope QSTAB($\bar{G}$).

Exercise 2: **4 points**

Find an explicit orthonormal representation of C_5, i. e., determine the vectors u_i and c corresponding to the "umbrella" mentioned in the lecture.

Exercise 3: **4 points**

Show that for any graph G the maximum degree $\Delta(G)$ plus one is an upper bound for the chromatic number $\chi(G)$.

Exercise 4: **4 points**

The *Mycielski graphs* M_k, $k \geq 2$ are inductively defined as follows:

$$M_2 := P_2 \quad \text{(path of length 2)}$$

$$V(M_{k+1}) := V(M_k) \cup \{u_i \mid i \in V(M_k)\} \cup \{w\}$$
$$E(M_{k+1}) := E(M_k) \cup \{u_i w \mid i \in V(M_k)\} \cup \{u_i v_j, v_i u_j \mid v_i v_j \in E(M_k =\}.$$

Prove the following:

a) The clique number $\omega(M_k)$ is 2, $k \geq 2$.

b) The coloring number $\chi(M_k)$ is k, $k \geq 2$.