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Exercise sheet 9

Deadline: Thursday, June 28th, 2007, 08:30 h in MA-313

Exercise 1: **4 points**

The *resource-constraint shortest path problem (RCSP)* is the following.

given: Graph $G = (V, E)$, start and target node s and t, distances $c\colon E \to \mathbb{R}_{\geq 0}$, resource consumption $w\colon E \to \mathbb{N}_0$, resource bound $W \in \mathbb{N}_0$.

goal: Find a shortest st-path w. r. t. to c that does not consume more than W resources.

Show that RCSP is NP-hard.

Hint: Use a reduction from the Knapsack problem.

Exercise 2: **4 points**

Modify the Joksch algorithm to not only compute the cost of a shortest path of length k, but also to return the shortest path.

Exercise 3: **4 points**

Consider the MIP model for the vehicle routing problem

$$\begin{aligned} \min \quad & \sum_{t\in\mathcal{T}_k} c(t)x_t \\ \sum_{t\in\mathcal{T}_k\colon j\in V(t)} & x_t = 1, \quad \forall j \in J \\ & x_t \in \{0,1\} \end{aligned} \tag{MIP-VRP}$$

and its relaxation

$$\begin{aligned} \min \quad & \sum_{t\in\mathcal{T}_k} c(t)x_t \\ \sum_{t\in\mathcal{T}'_k\colon j\in V(t)} & x_t = 1, \quad \forall j \in J \qquad \text{(MIP-VRP')} \\ & x_t \in \{0,1\} \end{aligned} \tag{1}$$

where $\mathcal{T}_k$ is the set of all tours starting and ending in the depot and visiting at most k customers and $\mathcal{T}'_k$ is the same set of tours, but with repeated visits to one customer allowed. Assume that the distance function of the VRP satisfies the triangle inequality.

Show that the objective values of (MIP-VRP) and (MIP-VRP') coincide and every optimal solution of (MIP-VRP) is also feasible for (MIP-VRP').

Exercise 4: **4 points**

A paper company produces large rolls of paper of width W. However, it sells only (much) smaller rolls. Given a demand of b_i rolls of width w_i $(1 \leq i \leq n)$, how many large rolls are needed to satisfy the demand?

This problem is known as the *cutting stock problem*. Find a MIP model that can be solved by column generation. What is the pricing problem?