

“Discrete Geometry” (Kombinatorische Geometrie I)

Summer Term 2008

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Classes:	Mon 8:15- 9:45	MA 841	
	Mon 10:30-12:00	MA 841	Start: April 14, 10:30
Tutorial:	Fri 8:15-9:45	MA 549	Start: April 18

The first, larger part of this course will give a quick but hands-on introduction to the theory of convex polytopes and their combinatorial properties. It will start with an extensive discussion and analysis of examples. On this basis we develop the fundamental combinatorial facts. Then we concentrate on 3-dimensional polytopes (rather well-understood) and on 4-dimensional polytopes (active field of research). A survey of f -vector theory will lead us to discuss “extremal” polytopes.

The later part of the course will also cover aspects of discrete geometry that go beyond the topic of convex polytopes: duality theory, arrangements and configurations, tilings, complexity bounds, as well as high-dimensional aspects.

Web page: <http://www.math.tu-berlin.de/Vorlesungen/SS08/DiskreGeom/>

Software: Analysis and visualization of examples using Polymake, and JavaView

Prerequisites: Basic linear algebra

Language: BMS Course: taught in English upon request.

Continuation: “Computational Geometry”, Winter term 08/09.

Outline/Contents:

0. First examples: 3-dimensional, regular
1. More examples: cyclic, stacked, hypersimplices, $0/1$, ...
2. The fundamental representation theorem
3. Polarity, face lattices and their combinatorics
4. 3-Dimensional polytopes, Steinitz’ theorem, circle packings
5. 4-Dimensional polytopes, Schlegel diagrams; fatness and complexity, examples
6. f -Vectors and flag vectors; shellability, UBT, g -theorem
7. Duality theory: Gale diagrams, non-rational polytopes; Rigidity and the LBT
8. Hyperplane arrangements and point configurations
9. Tilings, Delaunay triangulations, Voronoï diagrams, lattices
10. Complexity bounds: Incidence bounds, and the zone theorem
11. High-dimensional aspects: Rounding, measure concentration, ...
12. Discrete Geometry overview.

References:

- [1] GÜNTER M. ZIEGLER: *Lectures on Polytopes*, Graduate Texts in Math. **152**, Springer, New York, 1995/2001/2007.
- [2] BRANKO GRÜNBAUM: *Convex Polytopes*, Graduate Texts in Math. **221**, Springer, New York, New York, 2003. Second ed. prepared by V. Kaibel, V. Klee and G. M. Ziegler (original edition: Interscience, London 1967).
- [3] JIŘÍ MATOUŠEK: *Lectures on Discrete Geometry*, Graduate Texts in Math. **212**, Springer, New York, New York, 2002.

