Technische Universität Berlin
Fakultät II – Institut für Mathematik

Berlin Mathematical School
Sommersemester 2008

Discrete Geometry

(Kombinatorische Geometrie I)

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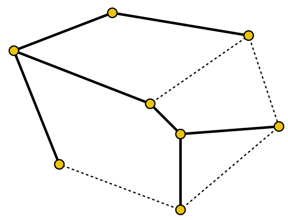
Exercise Sheet 5

Deadline: 26 May 2008

Exercise 22. **4 points**

Given a graph $G = (V, E)$, a *spanning tree* of G is a subgraph $T = (V, E')$ (that is $E' \subseteq E$) containing all vertices of G such that T is a tree (that is T is connected and contains no cycles). It is easy to see that every connected graph contains a spanning tree.



(a) Given a spanning tree $T = (V, E)$ of the graph of a 3-polytope P show that there is a spanning tree $T' = (V', E')$ of the graph of the polar polytope P^Δ with $|E| + |E'| = f_1(P)$.

(b) Show Euler's formula for 3-polytopes: $f_0(P) - f_1(P) + f_2(P) = 2$.
(*Hint:* Use the fact that a tree with n vertices has exactly $n - 1$ edges.)

Exercise 23. **4 points**

(a) Use Euler's formula to show that $f_0(P) \leq 2f_2(P) - 4$ and $f_2(P) \leq 2f_0(P) - 4$ for every 3-polytope P.

(b) Show that the Hirsch conjecture is true for 3-dimensional polytopes.

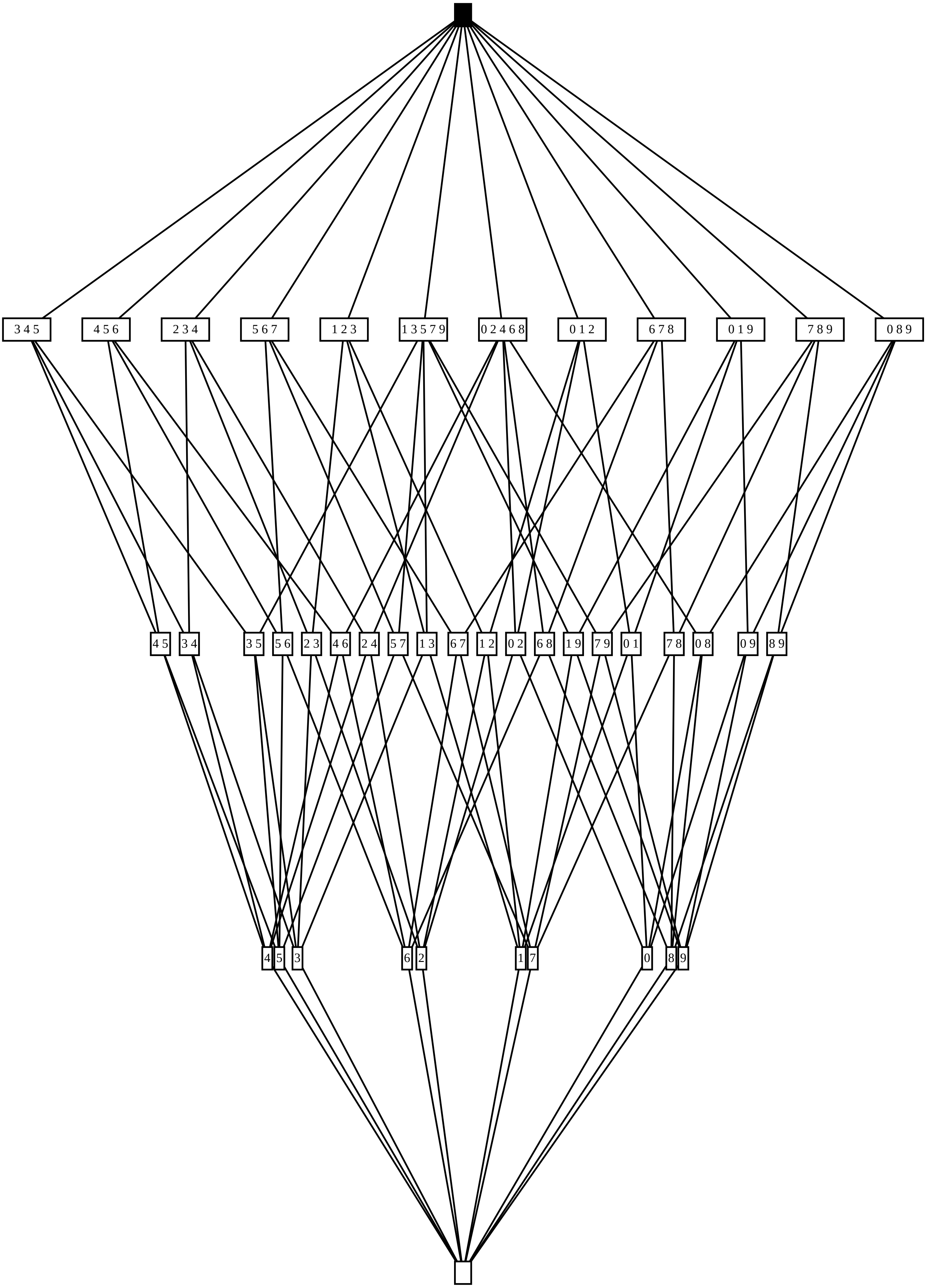
Exercise 24. **4 points**

Show that the shape of a quadrangular facet of a 3-polytope can be prescribed. More precisely: Given a 3-polytope P with a quadrangular facet, show that there is a combinatorially isomorphic 3-polytope P' where the corresponding facet is a unit square.

Exercise 25. **4 points**

Sketch the graph of the polytope given by the face lattice on the backside. Can you describe how to obtain a geometric realisation of the polytope? (It's called an *antiprism.*)

please turn over



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