Technische Universität Berlin
Fakultät II – Institut für Mathematik

Berlin Mathematical School
Sommersemester 2008

Discrete Geometry

(Kombinatorische Geometrie I)

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Exercise Sheet 6

Deadline: 2 Jun 2008

Exercise 26. **4 points**

Sketch Schlegel diagrams of the following 4-polytopes:

(a) A pyramid over a triangular prism, pyr bipyr Δ_2;

(b) The product of a 3-gon and a 6-gon;

(c) A 4-dimensional crosspolytope C_4^{Δ};

(d) A twice stacked 4-simplex.

Exercise 27. **4 points**

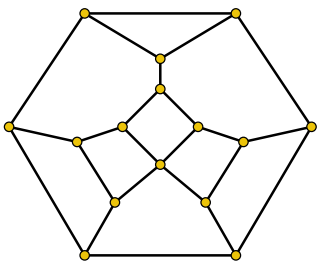
Let $P \subset \mathbb{R}^d$ be a d-polytope and $\mathbf{c} \in \mathbb{R}^d$ define a linear function $\mathbf{c}^\top \mathbf{x}$ (not necessarily in general position). Suppose the graph $G(P)$ of P carries the orientation induced by $\mathbf{c}$, with edges orthogonal to $\mathbf{c}$ ignored (or oriented randomly).
Show that for every vertex $\mathbf{v}$ of P that is not optimal with respect to the functional $\mathbf{c}^\top \mathbf{x}$ there is an outgoing edge from $\mathbf{v}$.

Exercise 28. **4 points**

Show that if P is a 3-polytope then either P or P^{Δ} contains a triangular facet.

Exercise 29. **4 points**

Use the `polymake` client `tutte-lifting` to obtain a Tutte embedding of the graph on the right.
Explain how this polytope can be obtained from a triangular prism.



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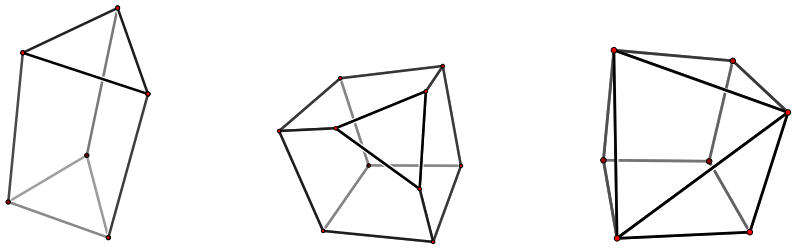
Exercise 30. **(Tutorial)**

In this exercise we compare the different proofs for Steinitz's theorem for the following examples:



a triangular prism, a cube with a vertex cut off and another cube with one vertex cut off "completely", i.e. by cutting with a hyperplane that goes through all 3 neighbouring vertices.

(a) How many simple Δ-Y-transformations do you need to get to a simplex? If only "cutting off a vertex" (that is, Y-Δ-transformations) are allowed, how often do you have to polarize on the way?

(b) Construct a (correct!) "rubber band" drawing that can be lifted to 3-space.

(c) Construct a (correct!?) planar circle packing representation.