

Discrete Geometry

(Kombinatorische Geometrie I)

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Exercise Sheet 9

Deadline: 23 Jun 2008

Exercise 41.

4 points

Use the g -theorem and the McMullen matrices M_4 and M_5 to obtain the g -vector of the following polytopes:

- (a) the 4-dimensional crosspolytope C_4^Δ ,
- (b) a 4-dimensional stacked polytope on 8 vertices,
- (c) a 4-dimensional cyclic polytope on 8 vertices,
- (d) the 5-dimensional crosspolytope C_5^Δ ,
- (e) a 5-dimensional stacked polytope on 10 vertices,
- (f) a 5-dimensional cyclic polytope on 10 vertices.

Exercise 42.

4 points

Prove the following lemma on the way to the lower bound theorem:

If $f_1 \geq df_0 - K_d$ holds for all simplicial d -polytopes, where K_d is a constant depending only on d (that is, not on the polytope), then the lower bound theorem holds for the edges, that is

$$f_1(P) \geq df_0(P) - \binom{d+1}{2}$$

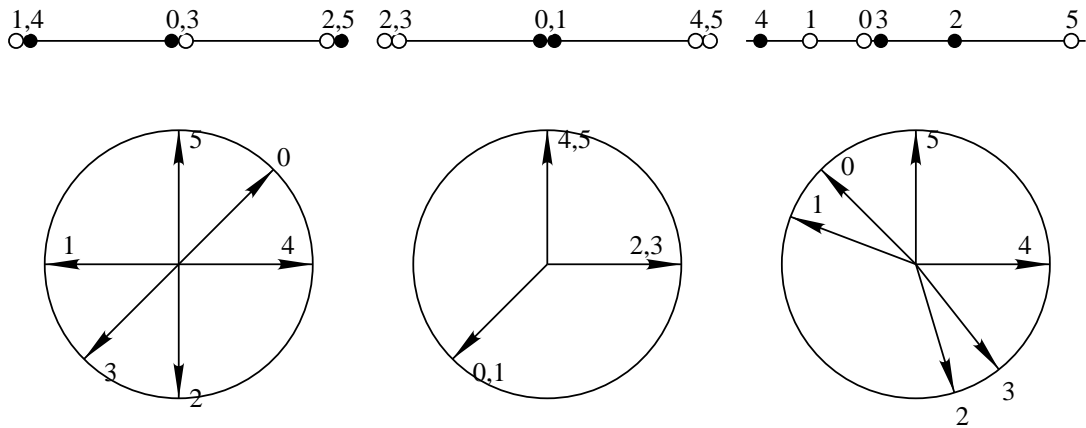
for all simplicial d -polytopes P .

(*Hint:* Consider a reflection of P in one of its facets.)

Exercise 43.

4 points

- (a) Consider the three Gale diagrams below. Determine the vertex-facet incidences of the corresponding polytopes. Which polytopes are described by the diagrams?



- (b) Let P be a d -polytope and G its Gale diagram. How does the Gale diagram change for the pyramid and the bipyramid over P ?

Exercise 44.

4 points

A *truncated icosahedron* F is obtained from a regular icosahedron (centered at the origin) by cutting off the 12 vertices by symmetric hyperplanes. More precisely: each hyperplane is orthogonal to the vertex vector it cuts off and the facets of F are regular hexagons and pentagons.

Calculate the f -vector of F . How long do the edges of F have to be such that F complies with the FIFA regulations?



(Hint: If you're not so sure about the regulations you might want to consider

http://www.fifa.com/mm/document/affederation/federation/laws_of_the_game_0708-10565.pdf)