Technische Universität Berlin
Fakultät II – Institut für Mathematik

Berlin Mathematical School
Sommersemester 2008

Discrete Geometry

(Kombinatorische Geometrie I)

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Exercise Sheet 10

Deadline: 30 Jun 2008

Exercise 45. **4 points**

State and prove a condition for the Gale diagram of a polytope that characterises k-simpliciality:

Proposition. A polytope is k-simplicial $\iff$ ...

Exercise 46. **4 points**

Let $P \subset \mathbb{R}^d$ be a polytope with $\mathbf{0}$ in its interior. Show that the face fan of P is the normal fan of P^Δ and vice versa.

Exercise 47. **4 points**

Enumerate all 3-dimensional zonotopes that are generated by 5 vectors in $\mathbb{R}^3$ and calculate their f-vectors and the numbers of simple vertices.

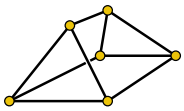
Exercise 48. **4 points**

Show that if every projection of a polytope P to $\mathbb{R}^3$ is a zonotope, then P is a zonotope itself.
Give an example that shows that the projections to $\mathbb{R}^2$ do not suffice for this.

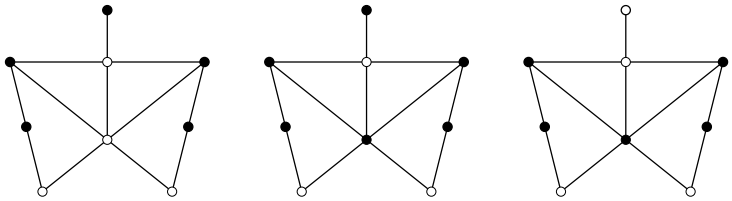
PLEASE TURN OVER

Exercise 49. **(Tutorial)**

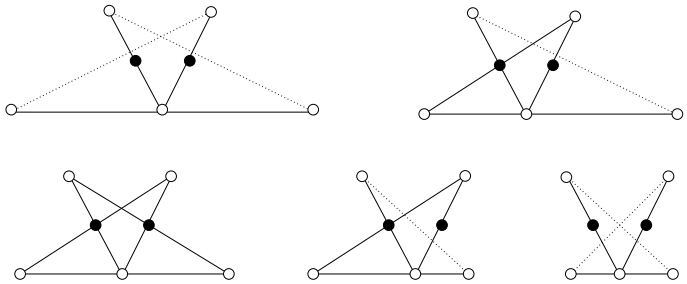
(a) Draw Gale diagrams of a pentagon and of the "tent" on the right. What happens to the Gale diagram if one of the two upper vertices is slightly "wiggled"?
What is the Gale diagram of a hexagon?



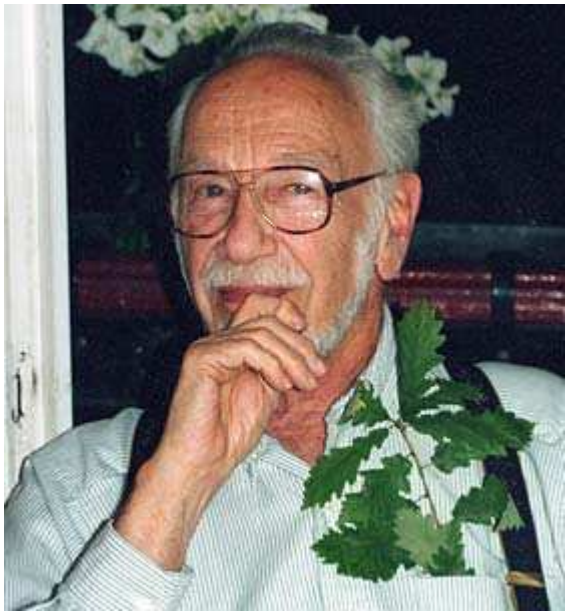
(b) Which of the following point configurations are affine Gale diagrams of polytopes? Why, or why not?



(c) Consider the following diagrams. Verify that they are affine Gale diagrams and construct the associated polytopes.



Hint: Consider the one you like best first and then describe how you have to modify your polytope in order to get the others.



David Gale (13 Dec 1921 – 7 Mar 2008)